A novel mechanism for tracking of the sun

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Abstract

There are many applications, which require the guidance of a body as if it is being rotated about an external point. One way to achieve such motion is by use of appropriately designed guide rails. The use of a four-bar mechanism to approximately achieve similar motion is also known in literature. In this paper, a novel mechanism has been presented, which can *exactly* rotate a link about an arbitrary external point while maintaining an arbitrary orientation of the link. The usefulness of such a mechanism is illustrated by the design of a solar concentrator where a parabolic mirror is attached to the link and the solar energy, throughout the day, is focused at a receiver, which is stationary.

1. Introduction

Parabolic concentrators direct the rays parallel to its principal axis on to the focal point and have been extensively used as solar collectors [1]. In most of the contemporary designs of these concentrators, the receiver is directly attached to the mirror and the whole assembly has to move while tracking the sun. This arrangement leads to design and dynamic problems, particularly when the receiver is heavy. A heavy receiver requires stronger backup structures, larger drives and expensive guide rails.. If the collector could be kept stationary, then the design of the concentrator, its backup structure and the drive mechanism could be made simpler and cheaper. In this arrangement, the mirror would have to move in a circle centered about the focus. To track the sun throughout the day and the year, the mirror would need to move in two different planes - a plane parallel to the horizon (azimuth) and in another plane perpendicular to it (elevation). While azimuthal rotation can be easily accomplished, it is difficult to rotate the mirror about focus in the perpendicular plane. In traditional designs, the mirror is guided along a circular track with the center at the focus making it very bulky and cumbersome [2]. The motivation of this work is to arrive at a simple mechanism that can perform the task of moving a parabolic mirror in the horizontal plane, throughout the day, in a way such that the sun's incident radiation is always focused at a chosen but arbitrary point.

From a mechanism point of view, the above described task comes under the topic of rigid body guidance problem in which the body is required to undergo rotation about an external point. The rigid-body guidance problem can be solved with a four-bar mechanism where the orientation of the body at few points on the coupler curve is specified [3,4]. However, this is only an approximate method and cannot be used for the exact tracking requirement in our case. There exist mechanisms for exactly tracing the circular path (like the peaucellier mechanism [5]) but they do not maintain the orientation of the body as required in our problem. Another approach is to use a three degree of freedom open kinematic chain such as a planar 3R manipulator [6]. However, this requires three actuators and complicated control, and moreover, since the problem is of one degree of freedom (the orientation and the position of the mirror are function of elevation angle only) a single degree of freedom mechanism would be more desirable for elevation tracking. In this paper, a novel single degree of freedom mechanism for planar rotation of a body about a fixed external point has been presented. This mechanism is similar to the Kemps mechanism. To achieve the tracking of the sun throughout the day and throughout the year this single degree of freedom mechanism is put on a rotary table whose rotation axis is

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perpendicular to the horizontal plane. The required azimuth and elevation angles at any instant of time can be obtained using the two actuators in the single degree of freedom mechanism and the rotary table.

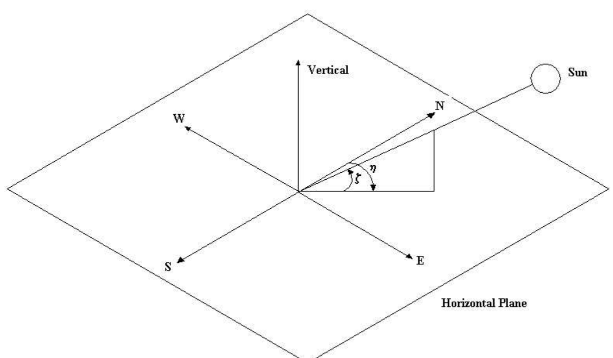
This paper is organized as follows: in section 2, a brief overview of the apparent motion of the sun as observed from Earth and an outline of a program written in Matlab to obtain the elevation and azimuth angles of the sun has been presented. In Section 3, the basic principle and the design of the novel single degree of freedom mechanism to track the elevation of the sun throughout the day has been presented. The design of the rotary table has also been presented. In section 4, some simulation results have been presented and in section 5, the conclusions and the scope for future work have been presented.

2. A brief overview of the motion of the sun

To design the linkage and the mechanism for tracking the motion of the sun, throughout the day and throughout the year, the elevation and azimuth angle of the sun at each instant of time need to be obtained. The following observations about the motion of the sun have been made:

1) The path of the Earth around the sun is approximately an ellipse with the sun at one of the focus. The exact geometry of the trajectory can be obtained from empirical formulas available in literature, which are based on astronomical observations [7].
2) The path of the sun across the sky is dependent on the latitude, longitude and altitude of the point on Earth.
3) The position of the sun in the sky, finally, depends on the time of the day, the month and the year.

A Matlab code has been written, which gives azimuth and elevation angles of the sun at any given time in GMT .It may be noted that the typical definition of the azimuth angle is the angle from the local North (in clockwise direction) of the projection of the line from the point to the sun on the horizontal plane. The elevation angle is defined to be the CCW angle made by the line collinear with the sunrays with the horizontal plane. These are shown in figure 1.



η= azimuth angle
ζ=elevation angle **Figure 1: Definition of azimuth and elevation angles**

The variation of the azimuth and the elevation angle for 4 days (two solstice, one equinox and one intermediate date) in 2005 for Bangalore is shown in figure 2. The output of the program matches with the data available from websites [8]. Roughly, speaking the range of elevation is between 0 and 90 degrees and the azimuth angle can vary between 0 and 360 degrees.

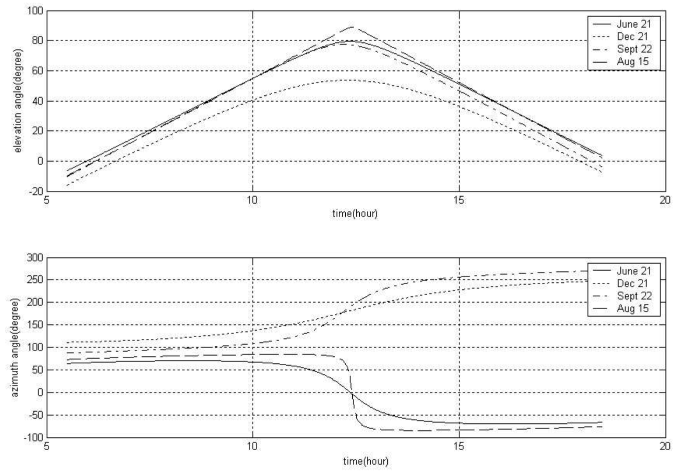


Figure 2: Plot of elevation and azimuth angles on four days

3. Design of the sun tracking mechanism

In this section, the design of the sun tracking mechanism has been presented. To design the sun tracking system, the azimuth and elevation angles as obtained from the program need to be tracked. It may be noted that a sensor-based approach can also be used. However, a sensor-based tracking is more complicated and can run into trouble in presence of clouds. In addition, since the accuracy requirement is not so high, a simple open-loop feed forward control with the desired angles obtained from a program can be used.

First a novel four bar based mechanism for elevation tracking is designed and then briefly the design of a rotary table has been discussed. The two, together, can track the path of the sun throughout the day and throughout the year.

Basic Principle

The basic principle of the proposed mechanism is based on the notion that for a closed loop mechanism, with some values of joint angles, a *similar* mechanism having *proportional link lengths* will have the same value of joint angles in the same configuration. As an example, consider the four-bar mechanism, denoted by ABCD with fixed joints at A and B, in figure 3. Consider another similar mechanism, EGDF, also shown in figure 3. If the angles θ and ϕ denote the input and output of the four bar ABCD, then one can prove by geometry that the angle $\angle$EGC and angle $\angle$GEF are respectively θ and ϕ.

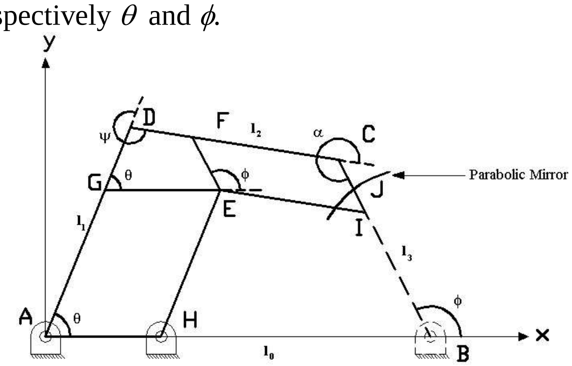


Figure 3: Basic four-bar mechanism with scaled links

The loop-closure equations of the four-bar ABCD are given by

$$X = l_0 + l_3 \cos(\phi) = l_1 \cos(\theta) + l_2 \cos(\theta+\psi) \quad (1)$$

$Y = l_3 \sin(\phi) = l_1 \sin(\theta) + l_2 \sin(\theta+\psi)$ (2)

From the equations, ϕ and ψ can be obtained as a function of θ and these are given by

$$\phi = 2\tan^{-1}\left(\frac{b \pm \sqrt{b^2 - c^2 + a^2}}{-c + a}\right) \qquad (3)$$

where

$a = 2 l_0 l_3 - 2 l_1 l_3 \cos(\theta)$, $b = -2 l_1 l_3 \sin(\theta)$, and $c = l_0^2 + l_1^2 - l_2^2 + l_3^2 - 2 l_0 l_1 \cos(\theta)$

Again from the loop closure equation

$M = l_2 \cos(\theta + \psi) = l_0 + l_3 \cos(\phi) - l_1 \cos(\theta)$ (4)

$N = l_2 \sin(\theta+\psi) = l_3 \sin(\phi) - l_1 \sin(\theta)$ (5)

Thus

$\psi = atan2(M,N) - \theta$ (6)

If ϕ and θ are related by equation (3), then point C will move on a circle with center B and the orientation of link CB is given by the angle α. It may be noted that

$\alpha = 3\pi + \phi - \psi - \theta$ (7)

Now consider, the joint B removed with link CB replaced by a smaller link CI. The open chain ADCI has three degrees of freedom. If point C still has to trace a circle centered at B and the link CI to maintain the orientation, then it must be ensured that ψ and α have values same as those obtained from equations (6) and (7). This can be achieved by using the notion of similar mechanisms discussed earlier.

From geometry, the angle $\angle$DGE must be θ and the angles α and ϕ, in the scaled down mechanism, must be same as in the original four-bar mechanism. Hence, if the joint at G can be rotated by θ, then the link DC will move exactly same as in the four-bar ABCD. This can be achieved by making AGEH a parallelogram linkage.

Likewise to maintain the orientation of link CI as in the original four-bar link CB, joint C must be rotated by the angle α obtained from the equation (7). Again by making EFCI a parallelogram linkage, this can be achieved. The sketch of the final mechanism, where point C rotates about point B and link CI maintains an orientation α, is shown in figure 3.

Degree of freedom of mechanism

The degree of freedom of the complete mechanism can be obtained by the Gruebler's equation is

$F = 3(N-1) - 2J - H$ (8)

Where, F is the degree of freedom of the mechanism, N is total number of links, J is the total number of one degree of freedom joints, and H is the number of higher pairs.

For this mechanism, $N = 8$, $J = 10$, $H = 0$ and hence $F = 1$.

Synthesis of the elevation tracking mechanism

With the basic concept of the mechanism shown in figure 3, analysis to obtain the dimensions of the links is now presented. The focal length and the diameter of the mirror are first chosen. These were chosen to be 2.0 m and 4.8 m respectively. The synthesis procedure can be easily modified for any other set of these values.

For the chosen focal length and mirror diameter, the minimum height H of the focus can be obtained by finding the radius of circle in which the extreme ends of the mirror rotates. From the equation of the parabola and using simple geometry (see fig. 4)

R = BK = sqrt(BL^2 + LK^2)

where BL is given by *(F – (D^2)/(16*F))* and KL is *0.5D* with *F* and *D* denoting the chosen focal length and diameter respectively. For the mirror to avoid hitting the ground, the height *H* must be greater than *R*.

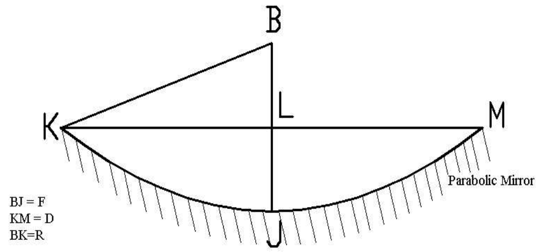


Figure 4: Extreme position of the parabolic mirror

Taking an arbitrary clearance of c_1, $H = R + c_1$.

With *H* determined, the actuation point at suitable distance *X* along the ground can further be chosen. As shown in figure 5a, *H* and *X* determines the length of the fixed link, and this gives l_0 as

$$l_0 = \sqrt{H^2 + X^2} \qquad (9)$$

Further from figure 3, the length BC is same as l_3. It may be noted that l_3 is related to the chosen focal length – if the mirror of chosen focal length, *F*, is placed on the link C I (see figure 5a), then $l_3=F+c_2$ where c_2 is arbitrary fraction of *F* determined by assembly and other requirements.

Thus the link lengths l_0 and l_3 of the basic four-bar are known and the optimum lengths of the input (l_1) and the coupler link (l_2) are left to be determined. The requirements are that the output link should move between the two specified positions (0^o and 90^o elevation) (figures 5a and 5b), and the input link should not move much below the horizontal level. Further the total length of input and the coupler link need to be optimized.

For this problem the parabolic mirror has to rotate about its focus point so as to track the elevation angle of the sun. The elevation angle of the sun varies in the range of approximately 0^o-90^o as seen from figure2. The input link of the basic four-bar should therefore move between two angles θ_1 and θ_2. These two limit positions can be obtained in the following manner:

1) Consider the mirror, at point between C and I, to be at the morning position. Choose the links AD and DC to be collinear. If it is not chosen as collinear then, the link CI (and the mirror) will first rise and then fall, thereby leading to unnecessary movement. This arrangement also ensures that it is an extreme position.
2) From the computed l_0 and l_3 and the fact that l_1 and l_2 are collinear, the four-bar mechanism at the morning position can be drawn. This is shown in figure 5a where it must be noted that JB is the focal length of the mirror. The other extreme position for elevation angle of 90 degrees is shown in figure 5b.

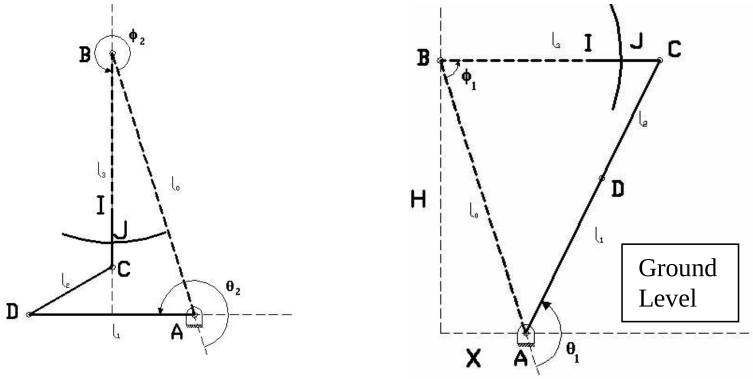


Figure 5: Extreme positions of mirror and the mechanism

Now the loop closure equations of a four-bar mechanism are used to obtain the values of the two missing link lengths, l_1 and l_2. For extreme position 1,

$$l_2 = \sqrt{(l_0 + l_1 cos\theta_1 - l_3 cos\phi_1)^2 + (l_1 sin\theta_1 - l_3 sin\phi_1)^2} \tag{10}$$

where ϕ_1 is the extreme value of the output angle and the corresponding input angle is θ_1 at extreme position 1. Like wise for the extreme position 2,

$$l_2 = \sqrt{(l_0 + l_1 cos\theta_2 - l_3 cos\phi_2)^2 + (l_1 sin\theta_2 - l_3 sin\phi_2)^2} \tag{11}$$

where ϕ_2 and θ_2 are the values of output and input, respectively, at extreme position 2. After eliminating l_2 from the above two equations

$$l_1 = \frac{-l_0 l_3(-cos\phi_2 + cos\phi_1)}{(l_0 cos\theta_2 - l_3 cos\theta_2 cos\phi_2 - l_3 sin\theta_2 sin\phi_2 - l_0 cos\theta_1 + l_3 cos\theta_1 cos\phi_1 + l_3 sin\theta_1 sin\phi_1)} \tag{12}$$

It may be noted that, for the given case, ϕ_1=0 and ϕ_2=90 degrees, and substituting these values in the above equation, an expression relating l_1, θ_1 and θ_2 is obtained. $\Delta\theta$ is defined as θ_2 - θ_1, and l_1 is plotted versus $\Delta\theta$. This is shown in figure 6.

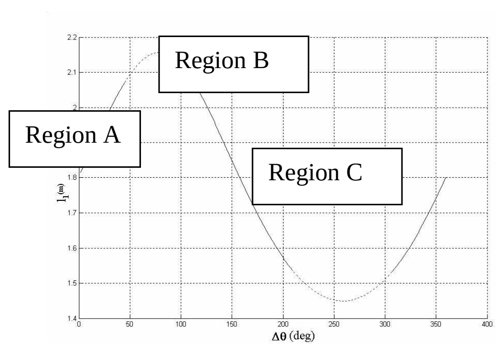


Figure 6: Possible solutions of l_1

The values of l_1 shown in the plot do not guarantee the mobility of the mechanism. To determine the mobility of the mechanism, Grashoff's criteria [5] can be used. However, in this case simple triangle inequalities have been used to obtain equivalent results

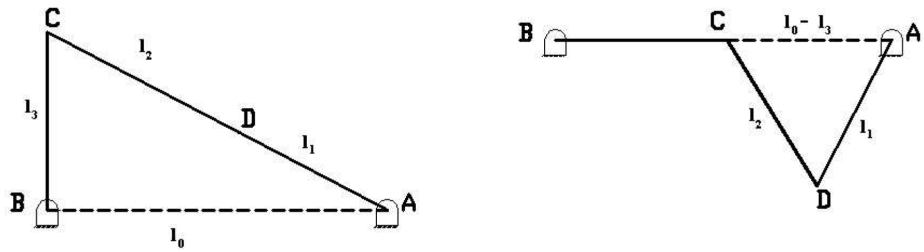


Figure 7: Criteria for mobility

Applying the triangle inequality in triangle ABC (see figure 7a), denoting the extreme position 1, $l_0 < l_1 + l_2 + l_3$, $l_1 + l_2 < l_0 + l_3$ and $l_3 < l_0 + l_1 + l_2$ (13)

Similarly applying triangle inequality in triangle ACD (see figure7b), denoting the extreme position 2, $l_0 < l_1 + l_2 + l_3$ is obtained. This inequality is same as the first inequality in equation (13). Another inequality is also obtained which is given by

$$l_2 < l_1 + l_0 - l_3 \quad (14)$$

and, finally,

$$l_1 < l_2 + l_0 - l_3 \quad (15)$$

The first set of inequalities, for the extreme position 1, is always satisfied. The inequalities in equations (14) and (15) can be used to eliminate some of the values of l_1 shown in figure 6. The possible regions, after elimination, are marked as A, B and C in figure 6.

In addition, from simulation, it was observed that values of l_1 in region A and C, the full range of the output angle ϕ was not obtained. Hence, the values of l_1 given in region B are only left. To minimize $\Delta\theta$, the minimum value of $\Delta\theta$ and the corresponding l_1 from region B is chosen. These are given by l_1=2.0538m and $\Delta\theta = 116^{o}$. The sum of l_1 and l_2 is known, and from this l_2 is obtained to be 1.552 m. This completes the synthesis of the basic four-bar mechanism.

The length of the smaller scaled down links is determined by a scaling factor. In this preliminary design, the scaling factor is chosen to be 0.2. However, the scaling factor can be systematically chosen using dynamics or other assembly or material criteria.

In summary, the following are the designed dimensions of the elevation tracking mechanism.

Mechanism dimensions

Imaginary output link (BC) = 2.50 m, fixed link (AB) = 3.04 m, input link (AD) = 2.05 coupler link (CD) = 1.55 m ,Scaling factor = 0.20,Length of the link CI = 0.50 m, link FE = 0.50 m, link GE = 0.61 m, link AH = 0.61 m, link EH = 1.64 m, link EI = 1.24 m

Design of the rotary table

The mechanism designed will track the elevation angle. The azimuthal tracking is accomplished by rotating the entire assembly about the axis passing through focus perpendicular to the horizontal plane. This is accomplished by a rotary table.

4. Simulation Results

To verify that the mechanism indeed tracks the sun throughout the day and throughout the year, the mechanism has been simulated in ADAMS [9] and PRO/E [10]. A few snapshots of the mechanism at different times of the day have been presented here. These are shown in figure 8a through 8c.

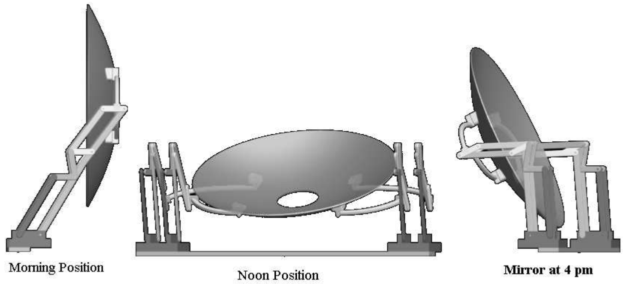


Figure 8: Three configurations of the sun tracking mechanism obtained from simulation

5. Conclusion

In this paper, a novel mechanism to track the elevation of the sun has been developed. The mechanism is based on a four-bar mechanism and use of scaled four-bar mechanisms and parallelogram linkages to drive the joints. The mechanism can be an alternate to guide rails used in traditional tracking antennas and parabolic dish reflectors. The design of the mechanism is presented in detail and a systematic method is presented to arrive at the link dimensions for elevation tracking of the sun. To track the sun throughout the day and throughout the year, the mechanism is used together with a rotary table. Simulation results demonstrate that the proposed system indeed achieve the desired objective.

Acknowledgement

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