Kinematics of pantograph masts

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Abstract

This paper deals with the kinematics of pantographs masts, which have widespread use as deployable structures in space. They are overconstrained mechanisms with degree-of-freedom (d.o.f), evaluated by the Grubler-Kutzbach formula, as less than one. In this paper, we present a numerical algorithm to evaluate the d.o.f of pantograph masts by obtaining the null-space of a constraint Jacobian matrix. In the process we obtain redundant joints and links in the masts. We also present a method based on symbolic computation to obtain the closed-form kinematic equations of triangular and box shaped pantograph masts and obtain the various configurations such masts can attain during deployment.

1.0 Introduction

Deployable masts used in space are prefabricated structures that can be transformed from a closed compact configuration to a predetermined expanded form in which they are stable and can carry loads. Deployable / foldable mast have one or more internal mechanisms [1-2] and their d.o.f as evaluated by the Grubler-Kutzbach criterion often turns out to be less than 1[3]. In this paper we study the kinematics of deployable masts made up of pantograph mechanisms or scissor like element (SLE). An SLE in two dimensional form has straight rods of equal length connected by pivots in the middle. The assembly has one d.o.f and the basic model can be folded and deployed freely. Three dimensional masts are created with SLE in such a way that they form a structural unit which in plan view is a normal polygon with each side being an SLE. The polygon can be equilateral triangle, square or normal n-sided polygon. By combining several of these normal polygon shaped units, structures of various geometric configurations can be created [4]. Active cables control the deployment and pre-stress the pantograph and passive cables are pre-tensioned in the fully deployed configuration. These cables have the function of increasing the stiffness when fully deployed. The whole system deploys synchronously.

The kinematics of pantograph masts can be studied by use of relative coordinates[5], reference point coordinates (as in the software package ADAMS) or Cartesian coordinates[6]. In this paper Cartesian coordinates, also called natural / basic coordinates, have been used. This method uses the constant distance condition for two or more basic points of the same link. Using unitary vectors the method can be extended to spatial mechanisms. The main advantage of using Cartesian coordinates is that the constraint equations are quadratic as opposed to transcendental equations, and the number of variables tends to be (on average) in between relative coordinates and reference point coordinates. In an earlier study, the foldability equations were formulated for SLEs based on geometric approach[7].

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The equations of motion for the SLE masts were obtained and solved numerically using Cartesian coordinates [8]. To the best of our knowledge, no attempt has been made by previous researchers in obtaining the closed-form solution for these masts. Closed-form equations, are expected to considerably reduce computation time and allow us to obtain different configurations a mast can attain which helps in better design of the system. In this paper the closed form kinematic equations are derived for the triangular and box mast using symbolic software MATHEMATICA[9].

Typical deployable masts have large number of links and joints. The d.o.f of these masts, as evaluated by the Grubler-Kutzback criteria, gives numbers less than one and hence, the d.o.f formula do not give a true number. Other methods such as screw theory and graph representation have been proposed by various researches to evaluate the correct d.o.f [10-11]. The concept of using first and higher order derivatives of constraint equations has been used for under constrained structural systems [12] to evaluate mobility and state of self stress. In this paper, we use the natural coordinates and the derivatives of the constraint equations to obtain the correct d.o.f of deployable masts. We also present an algorithm to identify redundant joints / links in a mast which leads to incorrect d.o.f from the Grubler-Kutzback criteria.

2.0 Kinematic description of the mechanism

The simplest planar SLE is shown in Figure 1. The revolute joint in the middle connect the two links of equal length. The assembly has one degree of freedom internal mechanism. The SLE remains stress free during the folding and extending process. The triangular mast and box mast are presented in Figure 2 and Figure 3 respectively.

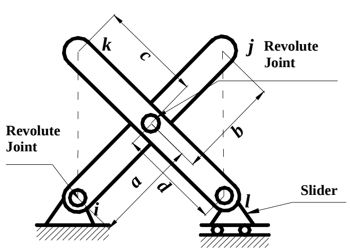

Figure. 1. Basic module of SLE

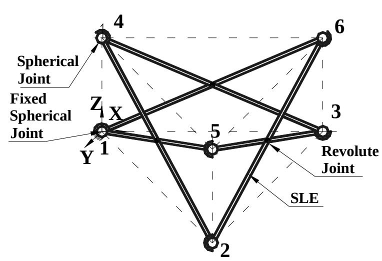

Figure 2. Triangular SLE mast

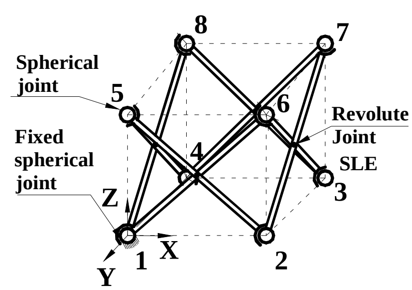

Figure 3. Box SLE mast

3.0 Kinematic Modelling

Modeling of three dimensional mechanisms with natural coordinates [6] can be carried out such that the links must contain sufficient number of points and unit vectors so that their motion is completely defined. A point shall be located on those joints in which there is a common point to the two links. A unit vector must be positioned on joints that have rotation or translational axis. All points of interest whose position are to be considered as a primary unknown variable can like wise be defined as basic points. In the natural coordinate system the constraint equations originate in the form of rigid constraints of links and kinematic joint constraints

3.1 Rigid constraints of link: This imposes a constant distance condition between two natural coordinates i and j of the link. This is given by $r_{ij} \cdot r_{ij} - L_{ij}^2 = 0$ (1)

where, $\boldsymbol{r_{ij}} = \{(X_i - X_j),(Y_i - Y_j),(Z_i - Z_j)\}^T$ and X_K, Y_K and Z_K , $K = i$ or j, are the coordinates at basic points i or j.

3.2 Joint constraints: These constraints describe the relative motion in accordance with kinematic joints that link them. The kinematic constraints corresponding to spherical joint is automatically satisfied when adjacent links share a basic point. Revolute joint is formed when two adjacent links share a basic point and an unit vector. The constraint equations can also be formulated for the slider pair[6].

3.2.1 Scissor like element : Scissor like element (SLE) or pantograph is shown in the Figure 1. Link $i\,j$ and $k\ell$ are coplanar and can rotate around the pivot. It is assumed that the two links are not equal and pivot is not in the middle. The position vectors must fulfill the following geometry conditions

$$\frac{b}{a+b}\boldsymbol{P}_i + \frac{a}{a+b}\boldsymbol{P}_j - \frac{d}{c+d}\boldsymbol{P}_k - \frac{c}{c+d}\boldsymbol{P}_l = 0 \qquad (2)$$

where, $\boldsymbol{P_m}$ and $m = i, j, k, \ell$ is the position vector consisting of coordinates of basic points.

3.3 Boundary constraints: The boundary constraints to exclude the global motion, need to be defined. If the basic point $\boldsymbol{P}$ is fixed, its coordinates are zero. If point $\boldsymbol{Q}$ moves along a plane perpendicular to Z axis, its Z coordinate is zero. These equations are written as

$$X_p = 0 = Y_p = Z_p = Z_q \qquad (3)$$

3.4 Constraint equations : The rigid constraint equations, joint constraints and boundary constraints can be written as $f_j(\mathrm{X}_1, \mathrm{X}_2, ...\mathrm{X}_n, t) = 0 \quad for \quad j = 1 \quad to \quad n_c$ (4)

in which n_c represents the total number of constraint equations including rigid body conditions, joint constraints due to SLE, slider, revolute pair and boundary constraints and n is the number of Cartesian coordinates of the system. Derivative of the constraint equations, with respect to time give the Jacobian matrix, which can be symbolically written as

$$[\boldsymbol{B}]\,\dot{\boldsymbol{X}} = 0 \qquad (5)$$

Since, equation (5) is homogeneous, one can obtain a non-null $\dot{\boldsymbol{X}}$ if the dimension of the null-space of $[\boldsymbol{B}]_{(n_c \times n)}$ is at least one. The existence of the null-space implies that the mechanism possess a d.o.f along the corresponding $\dot{\boldsymbol{X}}$ [6].

The deployable masts will have large number of kinematic pairs and links. It is useful to estimate the minimum number of kinematic pairs without losing the desired motion of the system. The dimension of null space basis is useful in estimating the d.o.f and identifying the redundant kinematic pairs.

3.5 Numerical Algorithm: The main steps in the numerical algorithm are as follows:

i) Add the derivative of the constraint equations one at a time in the following order
   - arising out of length constraints
   - arising out of Slider/SLE constraints

ii) At each step we evaluate dimension of null space of [***B***]. If dimension of null space of [***B***] doesn't decrease when a constraint is added it is redundant.

iii) Boundary constraints are added last and the dimension of null space of [***B***] is evaluated. If dimension of null space does not decrease after adding a boundary constraint, then corresponding constraint is redundant.

iv) The final dimension of the null space of [***B***] is the degree of freedom of the system.

In choosing the basic points the finite dimensions of joints are not taken into account. The spherical joints are taken at the intersection point of two adjacent links. In this formulation the initial folded configuration is not considered as it can have many singular configurations and hence does not give the true d.o.f of the system. The basic points of intermediate configuration is taken for the evaluation of Jacobian matrix.

4.0 Closed form solution for triangular mast

The numerical algorithm presented above does not give the closed-form expressions for direct and inverse kinematics of masts. To obtain them we have to use the original constraint equations (not in its derivative form) and attempt to obtain the minimal set of constraint equations and eliminate unwanted variables. Elimination of variables from a set of nonlinear equations is known to be an extremely hard problem and the difficulty increases with the number and complexity of each equation in the set. We have used a symbolic computation software, MATHEMATICA, to obtain closed-form solutions for some masts. The natural coordinates are useful in this respect since the equations are atmost quadratic in the variables used. In this section, we present the approach to obtain the closed-form solution for a triangular mast shown in Figure 2. For simplicity, we assume that (i) the links of SLEs are equal in length and the pivot is at the midpoint of the links, (ii) the joints 1 to 6 are spherical joints, (iii) the joints 1,2 and 3 are constrained to move in a plane, (iv) joint 1 is fixed, and (v) the links are rigid and the cables used for pre stressing does not affect the kinematics.

Using Equation (1), the length constraint equation for link 1-5 is given by

$$(X_5 - X_1)^2 + (Y_5 - Y_1)^2 + (Z_5 - Z_1)^2 - L_{15}^2 = 0 \qquad (6)$$

Similarly, the equations for the other links can be written.

The SLE equations can be obtained by using Equation (2) as

$$\mathbf{P}_1 + \mathbf{P}_5 - \mathbf{P}_4 - \mathbf{P}_2 = 0 \tag{7}$$

$$\mathbf{P}_2 + \mathbf{P}_6 - \mathbf{P}_3 - \mathbf{P}_5 = 0 \tag{8}$$

$$\mathbf{P}_3 + \mathbf{P}_4 - \mathbf{P}_1 - \mathbf{P}_6 = 0 \tag{9}$$

It can be observed that only two of the three linear SLE equations are independent. These can be checked by reducing these linear equations to row reduced echelon form. By using the assumptions (iii) and (iv), and substituting the above equations and observing that $X_4 = 0$ and $Y_4 = 0$ and solving, we get only three independent equations with five variables.

$$X_2^{\,2} + Y_2^{\,2} + Z^2 - L^2 = 0 \tag{10}$$

$$(X_3 - X_2)^2 + (Y_3 - Y_2)^2 + Z^2 - L^2 = 0 \tag{11}$$

$$X_3^{\,2} + Y_3^{\,2} + Z^2 - L^2 = 0 \tag{12}$$

Assuming X_2,Y_2 as known inputs, the solution for X_3 ,Y_3, *and* Z can be obtained as follows.

$$Z = \pm\sqrt{L^2 - (X_2^{\,2} + Y_2^{\,2})} \qquad X_3 = \frac{1}{2}\left(X_2 \mp \sqrt{3}Y_2\right) \quad Y_3 = \frac{1}{2}\left(Y_2 \pm \sqrt{3}X_2\right) \tag{13}$$

Hence, using Equations (13) and (7) through (9), the coordinates of all the joints of the triangular mast can be obtained in closed form. It can be observed that for the given X_2, Y_2 coordinates of joint 2, four configurations are possible-two configurations each for the positive and negative Z coordinate respectively. Each configuration is the mirror image of the triangle formed about the line joining the joints 2 and 1. Hence, assuming the mast moves only in the positive Z direction, the number of kinematic solution the mast can have is 2.

From the Equation(13), we have $L^2 - (X_2^{\,2} + Y_2^{\,2}) \geq 0$. If $L^2 - (X_2^{\,2} + Y_2^{\,2}) = 0$. The coordinates of joint 2 lie in a circle of radius L. This corresponds to the fully deployed configuration with $Z = 0$. Depending on the magnitude, the joint 2 moves along the circle of radius L. If $L^2 - (X_2^{\,2} + Y_2^{\,2}) < 0$, the solution for Z is imaginary. The coordinates of joint 2 lie out side the work space of the mast. The mast cannot reach these coordinates. If $L^2 - (X_2^{\,2} + Y_2^{\,2}) > 0$. The mast configuration includes the fully folded ($Z = L$) and very close to the fully deployed configuration, ($Z \approx 0$).

In the above analysis the independent variables are taken as the coordinates of joint 2. The above method can be used by taking the coordinates of joint 3 as independent variables. Alternatively

Equations (13) containing X_3 and Y_3 , can be simultaneously solved to evaluate the coordinates X_2 and Y_2 in terms of X_3 and Y_3 and Equation (12) can be used to evaluate the coordinate Z.

Box Mast : For the box mast, there are eight length constraint equations and nine independent SLE equations. We get following closed form solutions with X_2, Y_2 and X_3 as input. The solutions with positive Z axis are given in Table-1. This mast has eight solutions - four configurations each for positive and negative Z coordinate. Each configuration has two folded type and two deployed type of configurations.

Table 1 : Closed form solutions for the box mast $\left(P = \sqrt{2X_2X_3 - X_3^2 + Y_2^2}\right)$

Solution (i)	Solution (ii)	Solution (iii)	Solution (iv)
$Y_3 = Y_2 + P$	$Y_3 = Y_2 - P$	$Y_3 = Y_2 + P$	$Y_3 = Y_2 - P$
$X_4 = X_2$	$X_4 = X_2$	$X_4 = -X_2 + X_3$	$X_4 = -X_2 + X_3$
$Y_4 = Y_2$	$Y_4 = Y_2$	$Y_4 = P$	$Y_4 = -P$
$Z = +\sqrt{L^2 - (X_2^2 + Y_2^2)}$	$Z = +\sqrt{L^2 - (X_2^2 + Y_2^2)}$	$Z = +\sqrt{L^2 - (X_2^2 + Y_2^2)}$	$Z = +\sqrt{L^2 - (X_2^2 + Y_2^2)}$

5.0 Results and Discussions

In this section the methodology for evaluating the d.o.f described in previous section is used for triangular and box mast. Finally we present the deployment simulation of a single triangular mast using the closed from solutions.

5.1 Degree of freedom and redundancy evaluation: The triangular mast shown in Figure 2 has six rigid constraints, 3 SLEs and fixed boundary conditions at joint-1. The additional boundary conditions at points 2 and 3 are required to ensure the motion in XY plane. The results of null space are presented in Table 2. It is observed that the null space reduces by three by adding the first SLE. The null space reduces by only 2 for the 2nd SLE and it does not change for the 3rd SLE. The dimension of null space for the triangular mast is 2. Hence, it has one rigid body rotation about point 1 and one mechanism.

Table 2 : B matrix details for Triangular mast

Contents	Size of [***B***]	Null space	Remarks
Length constraints	(6,18)	12	
+ Boundary conditions	(11,18)	7	
+ SLE I	(14,18)	4	
+SLE II	(17,18)	2	(one component is redundant)
+ SLE III	(20,18)	2	SLE – III is redundant

The results of null space for box mast shown in Figure 3 are presented in Table 3. It is observed from the table that the null space reduces by three by adding each SLEs. The null space reduces by only 2 for the 3rd SLE. The null space does not change for the 4th SLE. The dimension of null space for the box masts is 3. Hence, the mast has one rigid body rotation about point 1 and two mechanisms. It is observed from the last row that the rank does not increase by adding additional boundary conditions because three boundary conditions are sufficient to represent the motion in a plane.

Table 3 : B matrix details for Box mast

Contents	Size of [***B***]	Null space	Remarks
Length constraints	(8,24)	16	
+ SLE I	(11,24)	13	
+SLE II	(14,24)	10	
+ SLE III	(17,24)	8	(one component is redundant)
+ SLE IV	(20,24)	8	SLE – IV is redundant
+ Boundary conditions ($X_1 = Y_1 = Z_1 = 0$)	(23,24)	5	
+ Boundary conditions ($Z_2 = Z_3 = Z_4 = 0$)	(26,24)	3	(Z_4 is redundant)

The analysis was also carried out for the hexagonal mast and similar behavoiur was observed. Hence, for the *n*-sided SLE mast, null space reduces by three for addition of each SLE and it reduces by only 2 for addition of *(n –1)*th SLE. The null space does not change for the addition of *n*th SLE.

5.2 Kinematic simulation for the triangular mast : The triangular mast with fully stowed configuration is taken as the initial configuration. The coordinate X_2 is varied from 0 to 30 units in steps of 2 units, Y_2 is taken as 0.0 and L = *30.0* . The equations (13) are solved to get the coordinates of joint 3 as the mast deploys. The simulation is shown in Figure 4. It is observed from the figure that as the joint 2 moves horizontally, the two solutions of joint 3 moves along the +60^o and –60^o line about X axis. The decrease in height of the mast during deployment is also shown. The simulation were also carried out for the two triangular masts attached at the sides. Due to page limitations these are not shown in this paper.

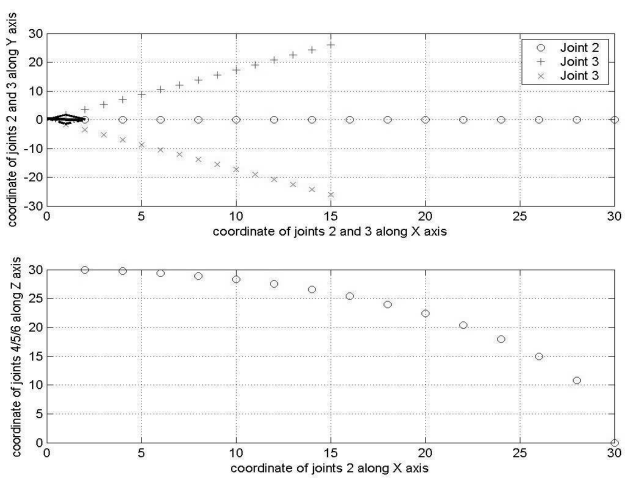

Figure 4 : Trajectory of joint coordinates for triangular mast

6.0 Conclusions

In this paper the Cartesian coordinate approach has been used to obtain the kinematic equations for the three dimensional deployable SLE masts. The d.o.f was evaluated using the Jacobian matrix. An algorithm was presented to identify the redundant kinematic pairs. It was observed that some of the SLEs were redundant. Hence, these masts can achieve the required single d.o.f with out these kinematic pairs. This formulation is easy to apply for the large number of masts. The kinematics of triangular and box masts were studied in closed form and the multiple solutions were evaluated. This method can be extended to masts of different shapes and for the stacked masts.

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