# Six component force-torque sensors using Gough-Stewart platform manipulators

Ashitava Ghosal*

**Abstract** – The six-degrees-of-freedom Gough-Stewart platform manipulator is a promising alternate architecture for the mechanical design of a six component force-torque sensor. Two basic configurations of the Gough-Stewart platforms are used for the design of six component force-torque sensors. In an *isotropic* configuration, equal sensitivity for all the six components of the force and torque being measured can be obtained. In a *singular* configuration, large mechanical magnification can be obtained for certain selected components of the force and torque, and, as a consequence, very small forces and/or torque along the selected directions can be measured. In this paper, we revisit the use of the Gough-Stewart platform manipulator as a sensor. Algorithms to determine the isotropic and singular configuration of the Gough-Stewart platform manipulator are presented. Two specific configurations of the Gough-Stewart platform with enhanced sensitivity to selected components of external forces and torque are taken up for analysis, design, fabrication and testing. Experimental results show that the prototype six component force-torque sensors can measure the external forces and torques as designed.

**Keywords: Stewart platform, force-torque sensor, isotropic and singular configurations, directionally sensitive**

*Dept. of Mechanical Engg., Indian Institute of Science,Bangalore, India. Email: asitava@mecheng.iisc.ernet.in

## 1 Introduction

The Gough-Stewart platform is a six-degree-of-freedom parallel manipulator. It was first proposed for the testing of wear in tyres by Gough [1] and later by Stewart as a flight simulator [2]. The Gough-Stewart platform manipulator consists of a *moving* platform connected to a *fixed* base by means of *six extendable* legs. Controlled extension and/or retraction of the legs results in six-degree-of-freedom motion of the moving platform. The main advantages of a Gough-Stewart platform manipulator is its increased load carrying capability since the load acting on the moving platform is 'shared' by the six legs. The Gough-Stewart platform manipulator has good positioning accuracy since the legs are primarily in tension/compression and undergo very little deflection due to bending. In addition, the errors in the actuators do not add up as in a serial manipulator – the positioning error is determined by the largest error in any single actuator. Due to these and other advantages the Gough-Stewart platform manipulator has been used in a variety of applications such as vehicle and flight simulators, accurate pointing devices, micro-motion devices and machine tools (see [3] and [4] and the references contained therein). The main disadvantages of parallel manipulators in general and Gough-Stewart platform manipulator in particular are the small workspace and singularities present in the workspace.

The Gough-Stewart platform can also be used as a six component force-torque sensor (see references [5]-[10] and the references contained therein). In a sensor configuration, the prismatic joints in the legs are replaced with sensing elements of different shapes, such as a ring [8] or 'H' slit beam [11], and mounted with strain gages. The sensing element can measure the axial force along the leg when an external force-torque acts on the top platform. In the case of a sensor, the small workspace is not a disadvantage as the moving platform essentially does not move. When used as a sensor, the friction, backlash and other non-linearities at the spherical and Hooke joint joints in the legs can affect the measurement in unpredictable ways. In order to avoid the friction and backlash at the joints, researchers have proposed the use of flexible hinges. The design of flexible hinges and its basic theory first appeared in Paros and Weisboard [12] and were refined by Zhang [13]. Flexible hinges has been used for the design of a precision six axis dynamometer [14] and, more recently, in a near-singular Stewart platform based force-torque sensor [10].

The Gough-Stewart platform in an *isotropic* configuration, as the name implies, has equal sensitivity for all components of force and torque [8]. Whereas a Gough-Stewart platform in a *near-singular* configuration will measure the six components of force and torque with enhanced sensitivity along selected directions [10]. Enhanced sensitivity is useful in many applications where it is known em a priori that some components of force or moment are much larger – in robotic assembly and manufacturing, the force in the normal direction is known to be 5 to 10 times larger than in the tangential directions[15]. In aerodynamics, it is known that the drag forces, pitching and other moments are typically 10 to 20 times smaller than the lift forces in a wing. In this paper, algorithms developed to obtain isotropic and singular configurations are presented. Design, analysis and testing of two near-singular configuration six component force-torque sensor are also presented.

This paper is organized as follows: in section 2, the geometry of Gough-Stewart platform is described and the equations related to the kinematics and static analysis of such platforms are presented. In section 3, the concept of isotropic configurations, algorithms and numerical results of Gough-Stewart platforms in isotropic configurations are presented. In section 4, the concept of a singular configuration, algorithms and results to obtain the singular configurations are presented. In section 5, two six component force-torque sensor in near-singular configurations, their design and experimental results demonstrating the concept of enhanced sensitivity for chosen components of force and torque are presented. The conclusions are presented in section 6.

## 2 Kinematics and statics of Gough-Stewart platform

Figure 1 shows a schematic of the Gough-Stewart platform manipulator. It consists of six extensible legs, denoted by $B_i - -P_i,\ i = 1, 2, .., 6$, with prismatic (P) joints in each leg. One end of a leg, $P_i$, is connected to the moving platform with spherical (S) joint and the other end, $B_i$, is connected to the fixed base with Hooke or universal (U) joint. The Gough-Stewart platform manipulator, in a general configuration, has six degrees-of-freedom – one can obtain arbitrary desired translatory and rotary motion along and about the $X$, $Y$ and $Z$ co-ordinate axis by appropriately actuating the six prismatic joints. The *direct kinematics* problem for a Gough-Stewart platform manipulator may be stated as follows: given the displacements at the six prismatic joints, obtain the position and orientation of the moving platform. The *inverse kinematics* problem is 'opposite' – given the position and orientation of the moving platform, obtain the displacements of the prismatic joints. The direct kinematics problem is known to be one of the hardest problems in robot kinematics. It involves eliminating the S and/or U joint variables and obtaining expressions for the position and orientation of the moving platform only in terms of the actuated prismatic joint variables. Several researchers have tried to solve this problem and eventually it was shown by Raghavan and Roth [16] that the required expression, after elimination, is a $40^{\text{th}}$ degree polynomial. This implies that there exists at most 40 possible configurations of the Gough-Stewart platform manipulator for given values of the prismatic (P) joint variables. The inverse kinematics problem on the other hand, as in all parallel manipulators, is much simpler. As shown in reference [17], the translation at the prismatic joint $l_1$ and the two rotations at the Hooke joint, $\psi_1$ and $\phi_i$, can be obtained as

$$\begin{aligned} l_1 &= \pm\sqrt{[(x,y,z)^T - {}^{B_0}\mathbf{b}_1]^2} \quad (1) \\ \psi_1 &= \mathrm{Atan2}(-Y, \pm\sqrt{X^2+Z^2}) \\ \phi_1 &= \mathrm{Atan2}(X/\cos\psi_1, Z/\cos\psi_1) \end{aligned}$$

where $(x,y,z)^T$ is the position vector of $P_1$ obtained from the given position and orientation of moving platform, ${}^{B_0}\mathbf{b}_1$ is the position of the point $B_1$ with respect to a origin in the fixed base, and $X, Y, Z$ are the components of the vector $[R(\hat{\mathbf{Z}},\gamma_1)]^T((x,y,z)^T - {}^{B_0}\mathbf{b}_1)$ with $[R(\hat{\mathbf{Z}},\gamma_1)]$ representing a constant rotation matrix. For the other legs, the above equations can be solved in 'parallel' and all the joint variables $l_i$, $\psi_i$, and $\phi_i$ for $i = 1, ..., 6$ can be found.

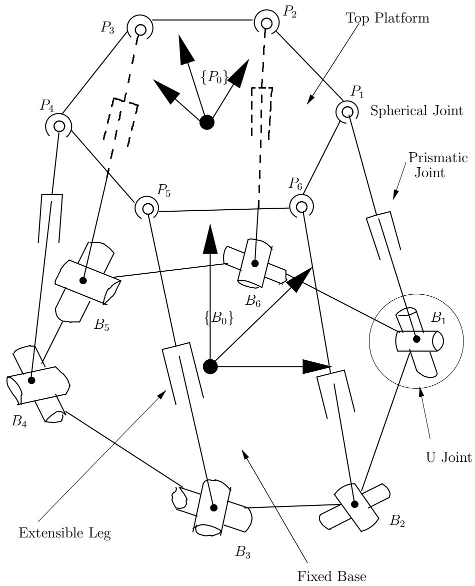


Figure 1: The Gough-Stewart platform

If an external force-moment is applied on the moving platform, one can obtain the *axial* forces in the legs required to keep the Gough-Stewart platform in equilibrium. This forms the topic of the statics of the Gough-Stewart platform manipulator and is well known (see, for example, Merlet [4]). Figure 2 shows an arbitrary $i^{\text{th}}$ leg and the vectors ${}^{B_0}\mathbf{b}_i$, ${}^{B_0}\mathbf{t}$, and ${}^{P_0}\mathbf{p}_i$ in the coordinate system $\{B_0\}$ and $\{P_0\}$ attached to the fixed platform and the moving platform, respectively. The figure also shows the prismatic joint whose translation along the leg vector $\mathbf{S}_i$ is denoted by $l_i$. The vector locating the moving platform point $P_i$ can be written with respect to the fixed base $\{B_0\}$ as

$${}^{B_0}\mathbf{p}_i = [R]^{P_0}\mathbf{p}_i + {}^{B_0}\mathbf{t} \quad (2)$$

where the rotation matrix $[R]$ transforms a vector in $\{P_0\}$ to the coordinate system $\{B_0\}$. The leg vector can be written as

$${}^{B_0}\mathbf{S}_i = [R]^{P_0}\mathbf{p}_i + {}^{B_0}\mathbf{t} - {}^{B_0}\mathbf{b}_i \quad (3)$$

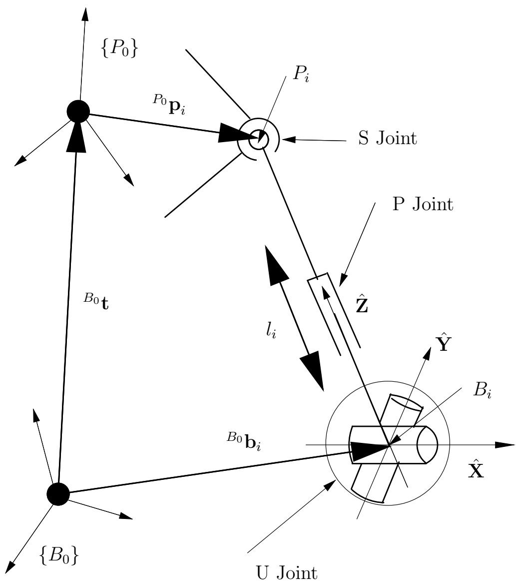


Figure 2: The Gough-Stewart platform

The axial force $f_i$ that can be exerted by the prismatic (P) joint acts along the unit vector ${}^{B_0}\mathbf{s}_i$ given by ${}^{B_0}\mathbf{S}_i/l_i$. The resultant force ${}^{B_0}\mathbf{F}$ and moment ${}^{B_0}\mathbf{M}$ that can be obtained by the application of the six $\mathbf{f}_i$'s are given by

$$\begin{aligned} {}^{B_0}\mathbf{F} &= \sum_{i=1}^{6} f_i {}^{B_0}\mathbf{s}_i \\ {}^{B_0}\mathbf{M} &= \sum_{i=1}^{6} f_i({}^{B_0}\mathbf{b}_i \times {}^{B_0}\mathbf{s}_i) \quad (4) \end{aligned}$$

Dropping the leading superscripts for convenience, the above equations can be written in a compact matrix form as

$$\begin{pmatrix} \mathbf{F} \\ \mathbf{M} \end{pmatrix} = [H]\mathbf{f} \tag{5}$$

where the $i^{\text{th}}$ column of the matrix $[H]$ denoted by $\mathbf{H}_i$ is given by

$$\mathbf{H}_i = \begin{pmatrix} \mathbf{s}_i \\ \mathbf{b}_i \times \mathbf{s}_i \end{pmatrix} \tag{6}$$

and $\mathbf{f}$ is $6 \times 1$ vector of axial forces applied by the prismatic (P) joints.

The matrix $[H]$ is called the wrench transformation matrix and can be used to obtain the external force and moment that can be supported by a set of known leg forces. To obtain the required leg forces $f_i,\ i = 1, 2, .., 6$ that is required to support a given $\mathbf{F}$ and $\mathbf{M}$ combination, the inverse of $[H]$ can be used. For a sensor application, the measured quantities are $f_i,\ i = 1, 2, ..., 6$ and as shown in equation (5), multiplying the $[H]$ matrix with $\mathbf{f}$ directly gives the unknown six components of the external load $\mathbf{F}$ and $\mathbf{M}$. It maybe noted that the computed $\mathbf{F}$ and $\mathbf{M}$ from equation (5) are the forces and moments with respect to the origin of the fixed base coordinate system $\{B_0\}$. If the force and moment vectors acting on the moving platform $\{P_0\}$ are given, they can be transformed to $\{B_0\}$ by using a transformation between $\{P_0\}$ and $\{B_0\}$, and this transformation can be obtained from the geometrical configuration of the Gough-Stewart platform. In a similar manner, if the force and moment acting on the moving platform are needed to be measured, an appropriate transformation between $\{B_0\}$ and $\{P_0\}$ need to be used.

The matrix $[H]$ forms the basis of the analysis of *isotropic* and *singular* configurations of a Gough-Stewart platform manipulator or sensor and this is discussed in the next two sections.

# 3 Isotropy in Gough-Stewart platform

The term 'isotropy' implies same or identical 'properties' in all directions. A large amount of literature exists on *kinematic* isotropy where the goal is to design Gough-Stewart platforms which can move *equally* well in *all* spatial directions [18, 19, 20, 21]. In the context of *force-moment* isotropy in Gough-Stewart platform, it implies the ability of platform to resist *external forces and moments* equally well in all spatial directions [18]. Intuitively, it would be desirable to have a isotropic configuration for a Gough-Stewart platform based six component force-torque sensor since the measurements would be equally sensitive or accurate for all the six components of the externally acting force and moment. Mathematically, isotropy implies that the eigenvalues (or singular values) of the $[H]$ matrix are all identical. Unfortunately, this does not have much physical significance since the elements of the top and bottom $3 \times 6$ sub-matrices of $[H]$ have *different* units – $\mathbf{s}_i$ has no units whereas $\mathbf{b}_i \times \mathbf{s}_i$ has units of length, and the elements of the bottom $3 \times 6$ sub-matrix will change with different choice of length units. Instead of evaluating the singular values of the full $[H]$ matrix, the singular values of the top and bottom $3 \times 6$ sub-matrices are considered in the work by Bandyopadhyay and Ghosal [22]. They introduce the concept of *combined* isotropy where a) the three singular values of the top $3\times6$ sub-matrices of $[H]$ are equal, b) the three singular values of bottom $3 \times 6$ sub-matrix are also equal, but c) the two sets of singular values are *not* necessarily identical. This approach leads to a more consistent mathematical treatment of the force-moment isotropy and the design of Gough-Stewart platforms for force-moment isotropy. The key steps and some of the main results in reference [22] are presented next.

The equations in (5) can be written separately as

$$\begin{aligned} \mathbf{F} &= [H]_{\mathbf{F}}\mathbf{f} \\ \mathbf{M} &= [H]_{\mathbf{M}}\mathbf{f} \end{aligned} \tag{7}$$

where $[H]_{\mathbf{F}}$, $[H]_{\mathbf{M}}$ are the top and bottom $3 \times 6$ sub-matrices of $[H]$, representing the $\mathbf{F}$ and $\mathbf{M}$ equations, respectively. Following standard matrix theory [23], the extreme values of $\mathbf{F}$ and $\mathbf{M}$ can be obtained from two eigenproblems

$$\begin{aligned} [g]_{\mathbf{F}}\mathbf{f} &= \lambda_{\mathbf{F}}\mathbf{f} \\ [g]_{\mathbf{M}}\mathbf{f} &= \lambda_{\mathbf{M}}\mathbf{f} \end{aligned} \tag{8}$$

where $[g]_{\mathbf{F}}$, $[g]_{\mathbf{M}}$ are the square matrices $[H]_{\mathbf{F}}{}^T\ [H]_{\mathbf{F}}$

and $[H]_{\mathbf{M}}{}^T\ [H]_{\mathbf{M}}$, respectively. These eigenproblems have the following characteristics (see [22] for details):

- The eigenvalues $\lambda_{\mathbf{F}}$ and $\lambda_{\mathbf{M}}$ are real and nonnegative.

- At the most three of the eigenvalues are nonzero in each case as the rank of $[H]_{\mathbf{F}}$ or $[H]_{\mathbf{M}}$ cannot exceed three.

- The eigenvalues can be solved in closed-form since we need to solve for at most a cubic polynomial. For example, the characteristic equations of $[g]_{\mathbf{F}}$ are

$$0 = \begin{cases} \lambda_{\mathbf{F}}^2 + a_1\lambda_{\mathbf{F}} + a_2, & n = 2 \\ \lambda_{\mathbf{F}}^3 + a_1\lambda_{\mathbf{F}}^2 + a_2\lambda_{\mathbf{M}} + a_3, & n = 3 \\ \lambda_{\mathbf{F}}^{n-3}(\lambda_{\mathbf{F}}^3 + a_1\lambda_{\mathbf{F}}^2 + a_2\lambda_{\mathbf{F}} + a_3), & n > 3 \end{cases} \tag{9}$$

  where $a_i$ are real co-efficients. The characteristic equations of $[g]_{\mathbf{M}}$ has exactly the same form as above.

- For a constraint of the form $\mathbf{f}^T\mathbf{f} = 1$, the tip of the force vector $\mathbf{F}$ lies on an ellipsoid in $\mathbf{R}^3$. The extreme values of $\mathbf{F}$, corresponding to the magnitude of the semi-major, semi-intermediate and semi-minor axis of the ellipsoid, are the square roots of the three non-zero eigenvalues of $[g]_{\mathbf{F}}$. Similarly, the the tip of the moment vector $\mathbf{M}$ lies on another ellipsoid in $\mathbf{R}^3$ and the extreme values of $\mathbf{M}$ are related to the three non-zero eigenvalues of $[g]_{\mathbf{M}}$. Mathematically, we can write

$$\begin{aligned} \lambda_{\mathbf{F}_i} &= \|\mathbf{F}^*\|^2 \\ \lambda_{\mathbf{M}_i} &= \|\mathbf{M}^*\|^2 \quad i = 1, \ldots, n \end{aligned} \tag{10}$$

  where $(\cdot^*)$ indicates the extreme value of a quantity.

  The concept of force and moment ellipsoids are similar to the concept of velocity ellipsoid for the motion of an end-effector presented in reference [17].

- The condition for force isotropy, $\mathbf{F} - isotropy$, can be written in terms of co-efficients $a_i$. They are

$$\begin{array}{ll} a_1^2 - 4a_2 = 0, & n = 2 \\ \left.\begin{array}{l} 3a_2 - a_1^2 = 0 \\ 27a_3 - a_1^3 = 0 \end{array}\right\} & n \geq 3 \end{array} \tag{11}$$

  In terms of the force ellipsoid, at $\mathbf{F} - isotropy$, the ellipsoid becomes a sphere.

- Similar to the above, the condition for moment isotropy, $\mathbf{M} - isotropy$, can be written as

$$\begin{array}{ll} b_1^2 - 4b_2 = 0, & n = 2 \\ \left.\begin{array}{l} 3b_2 - b_1^2 = 0 \\ 27b_3 - b_1^3 = 0 \end{array}\right\} & n \geq 3 \end{array} \tag{12}$$

  where $b_i$'s denote the real co-efficients of the characteristic equation corresponding to the eigenproblem associated with $\mathbf{M}$. In terms of the moment ellipsoid, at $\mathbf{M} - isotropy$, the ellipsoid becomes a sphere.

- Finally, the *combined* isotropy of force and moment can be written as

$$\left.\begin{array}{l} a_1^2 - 4a_2 = 0 \\ b_1^2 - 4b_2 = 0 \end{array}\right\} \quad n = 2 \tag{13}$$

$$\left.\begin{array}{l} 3a_2 - a_1^2 = 0 \\ 27a_3 - a_1^3 = 0 \\ 3b_2 - b_1^2 = 0 \\ 27b_3 - b_1^3 = 0 \end{array}\right\} \quad n \geq 3 \tag{14}$$

The above results can be used to compute numerical results for isotropy in Gough-Stewart platforms. It maybe noted that this algorithm gives *exact* isotropy as opposed to *approximate* isotropy used by some researchers (see, for example, [8]), and in this sense is a significant improvement.

### 3.1 Numerical example of combined isotropy in Gough-Stewart platform

The semi-regular Stewart platform manipulator (SR-SPM) is a special case of the general $6-6$ Gough-Stewart platform manipulator. It is the most widely used configuration in industry. In an SRSPM, the moving platform and the fixed base can be circumscribed in two circles of radii $r_t$ and $r_b$ centered at $P_0$ and $B_0$, respectively. Moreover there is a 3-way symmetry in an SRSPM. The angle subtended by the arcs $P_1 - P_2$, $P_3 - P_4$ and $P_5 - P_6$ at the centre $P_0$ are equal and denoted by $\gamma_t$. Likewise the angle subtended by the arcs $B_1 - B_2$ and $B_3 - B_4$ and $B_5 - B_6$ at centre $B_0$ are also equal and denoted by $\gamma_b$ (see figure 1). As shown in reference [22], with the moving platform kept horizontal at a height of $z = 0.4627$ and rotated about $Z$ axis by an angle $\pi/18$, the SRPSM will exhibit combined force-moment isotropy when $r_b = 1$, $\gamma_b = \pi/5$, $\gamma_t = \pi/10$ and $r_t = 0.3789$. The SRSPM in the combined isotropic configuration is shown in figure 3.

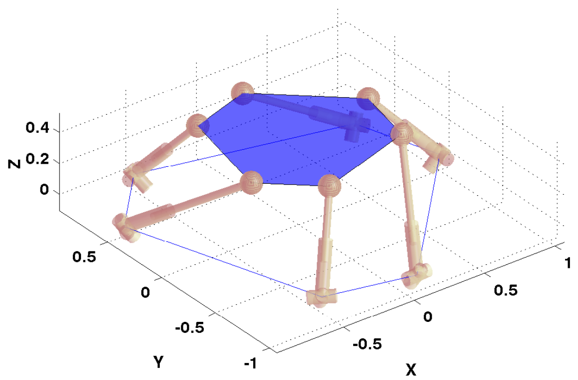


Figure 3: Combined force-moment isotropic configuration for an SRSPM (from [22])

# 4 Singularity in Gough-Stewart platform

The singular configuration for a Gough-Stewart platform manipulator is obtained when $\det[H] = 0$. From equation (5), we can write

$$\mathbf{f} = [H]^{-1}(\mathbf{F};\mathbf{M})^T \tag{15}$$

and if $\det[H] \rightarrow 0$, a finite $(\mathbf{F};\mathbf{M})^T$ will give rise to infinite forces in one or more legs, and some component(s) of the external force-moment $(\mathbf{F};\mathbf{M})$ cannot be supported by the leg forces. The eigenvectors corresponding to the zero eigenvalues of $[H]$ when mapped to $(\mathbf{F};\mathbf{M})^T$ give the singular directions and Gough-Stewart platform cannot withstand any force/moment applied along the singular directions. If the Gough-Stewart platform is in a *near-singular* configuration with $\det[H]$ small, then a *small* force/moment along the singular direction will give rise to *large* axial forces along the legs. This *mechanical magnification* can be used to design sensitive six component force-torque sensors where small forces/moments along certain directions can be more easily measured. This key concept was used in reference [10] to design a force-torque sensor with enhanced sensitivity to forces along $X$ and $Y$ directions and to moments about $Z$ direction. The algorithm to determine the singular direction for a Gough-Stewart platform and some of the key results from reference [10] are described next.

As discussed in section 3, the rank of $[g]_{\mathbf{F}}$ is at most 3 and three eigenvalues of $[g]_{\mathbf{F}}$ are always zero. At a $\mathbf{F}-$*singularity*, one ( or more) of the three non-zero eigenvalues of $[g]_{\mathbf{F}}]$ becomes zero and the force ellipsoid degenerates to a ellipse, a line or a point (similar to the concept of singularities of point trajectories presented in reference [24]). The singular directions of a Gough-Stewart platform for externally applied $\mathbf{F}$ can be obtained by mapping the eigenvectors corresponding to the zero eigenvalues of $[g]_{\mathbf{F}}$. Likewise starting with $[g]_{\mathbf{M}}$, we can obtain the singular directions corresponding to $\mathbf{M}$. In reference [10], the singular directions for several *regular* $6-6$ hexagonal Gough-Stewart platforms, with identical fixed and moving platforms, are presented. Two of these configurations are given by the following sequence of connection between the base points $B_i$ and moving platform points $P_i$.

- Configuration 1: $B_1 - P_1$, $B_2 - P_2$, $B_3 - P_3$, $B_4 - P_4$, $B_4 - P_4$, $B_5 - P_5$. In this configuration, the legs are *vertical.* Two of the three non-zero eigenvalues of $[g]_{\mathbf{F}}$ are found to be zero and the singular directions of force are along $X$ and $Y$ and the singular direction of $\mathbf{M}$ is about $Z$. In this configuration, the Gough-Stewart platform will not be able to withstand any force applied in the horizontal plane and any moment applied about the vertical axis.

- Configuration 2: $B_1 - P_2$, $B_2 - P_3$, $B_3 - P_4$, $B_4 - P_5$, $B_5 - P_6$, $B_6 - P_1$. In this configuration, the legs are *skewed.* All the three non-zero eigenvalues of $[g]_\mathbf{M}$ are found to be zero and $[g]_\mathbf{F}$ is of rank 3. In this configuration, the Gough-Stewart platform will not be able to withstand any of the three moment components.

At a singular configuration $\mathbf{f} \to \infty$. This is not feasible for an actual sensor application since the axial forces can be come too high and the sensor may fail. At a *near-singular* configuration, one can still get large axial forces in the legs and a reasonable mechanical amplification. In reference [10, 25], extensive simulations are presented, based on rigid and flexible models using Matlab [26] and NISA [27], to arrive at detailed design of the near-singular configurations which gives acceptable amplification and is still able to bear the external loads. These two sensors and some of the results obtained from experiments with them are presented in the next section.

# 5 Two sensors based on near-singular Gough-Stewart platforms

In this section, we present the detailed design of two six component force-torque sensors and experimental results obtained from the hardware. We start with Configuration 1 which is sensitive to $F_x$, $F_y$ and $M_z$.

## 5.1 Configuration 1

To arrive at a near-singular configuration, the half-angle between any two connection points on the platform, namely $\gamma_t$ is perturbed from the nominal value of 30°. It is observed that if the half-angle deviates from the nominal by ±3° then the the condition number of $[H]$ is approximately 1910[1] with the length units chosen to be milli-meters. This indicates that there will be significant amplification. Based on this analysis, the nominal coordinates of $B_i$ and $P_i$ points are given in Table 1. It maybe noted that centre of the moving platform is chosen to be 100 mm above the centre of the fixed platform. Each leg is modeled as a thin rod with a ring shaped sensing element mounted with strain gauges and flexible hinges. The leg is made of titanium for its high strength, low weight and its ability to undergo large strain without failure. A finite element model of the sensor was made and analysed in NISA [27]. For a typical external loading of $F_x = F_y = F_z = 0.98$ N, $M_x = M_y = M_z = 49.05$ N-mm, the maximum deformation obtained from the finite element analysis was found to be 0.5 mm – see figure 4 for the undeformed (blue) and deformed (red) Gough-Stewart platform. The maximum stress level is about 294 N/mm$^2$ (see figure 5) at the flexible hinges. These values are well within the allowable values for the chosen material.

Table 1: Nominal geometry of Configuration 1 sensor

| Base coordinates | | | Platform coordinates | | |
|---|---|---|---|---|---|
| Point No. | $x$ mm | $y$ mm | Point No | $X$ mm | $Y$ mm |
| $B_1$ | 43.30 | 25.0 | $P_1$ | 41.93 | 27.23 |
| $B_2$ | 0 | 50.0 | $P_2$ | 2.62 | 49.93 |
| $B_3$ | -43.30 | 25.0 | $P_3$ | -44.55 | 22.70 |
| $B_4$ | -43.30 | -25.0 | $P_4$ | -44.55 | -22.70 |
| $B_5$ | 0 | -50.0 | $P_5$ | 2.62 | -49.93 |
| $B_6$ | 43.3 | -25.0 | $P_6$ | 41.93 | -27.23 |

A prototype of the designed force-torque sensor was made and is shown in figure 6. The prototype sensor was loaded externally, in a specially designed fixture, by means of standard dead weights. The loading and unloading was done in steps and limited to 0.98 N and 49.05 N-mm, respectively. The strain values were measured for each leg. Using a calibration done for each leg (see [10] for details), the strain measurements were converted to axial forces along

[1] The condition number of a matrix is the ratio of the magnitude of the largest to the smallest eigenvalue. At a singular configuration the condition number will be infinity and at an isotropic configuration, the condition number is 1.0 [23].

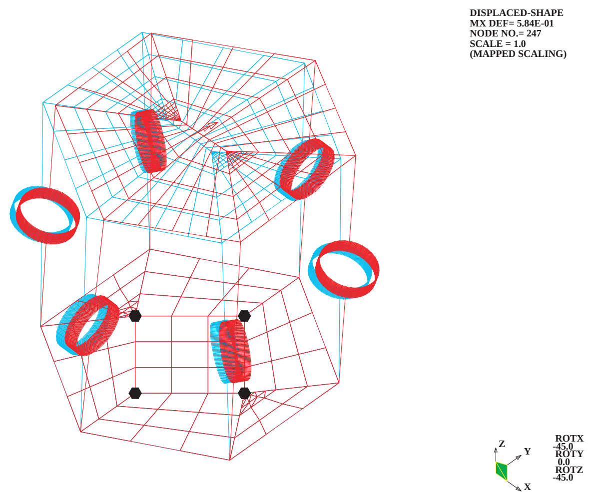


Figure 4: Deflection of the sensor in mm (from [10])

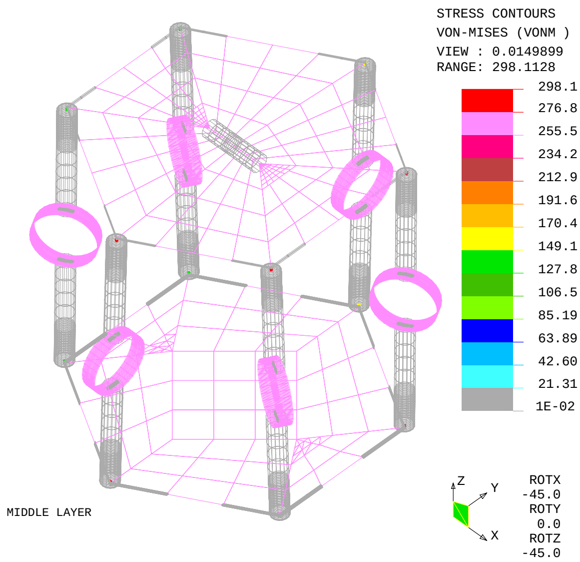


Figure 5: Stress in the sensor in $N/\mathrm{mm}^2$(from [10])

the legs and plots of applied load versus axial forces in each leg were plotted. The experimental data was compared with numerical results obtained from finite element analysis, and it was observed that the maximum error between the numerical and experimental data was less than 10% in the sensitive directions. It was also clear that the prototype sensor is sensitive to $F_x$, $F_y$ and $M_z$ and *not* to $F_z$, $M_x$ and $M_y$.

Using several measurements the elements of the $[H]$ matrix was computed using a least-squares technique [10]. The columns $\mathbf{H}_i$, $i = 1, 2, .., 6$ (see equation (6)) were obtained as $(-0.0204\ 0.0279\ 0.8890\ 22.7237\ -6.7289\ 1.3319)^T$, $(0.0273\ -0.0082\ 0.8294\ 44.3631\ -5.5169\ -1.5084)^T$, $(-0.0266\ -0.0367\ 0.8321\ 21.0266\ -5.0906\ 1.8969)^T$, $(-0.0210\ 0.0292\ 0.8845\ -18.6015\ -4.8826\ -1.4110)^T$, $(0.0380\ 0.0048\ 0.9704\ -45.1386\ -5.1129\ 1.2823)^T$, and $(-0.0117\ -0.0272\ 0.9712\ -26.4990\ -6.4894\ -1.9917)^T$. It maybe noted that the condition number of $[H]$ given above is about 1360 as against 1910 obtained for the nominal design. The $[H]$ matrix computed above now can be used to measure unknown $\mathbf{F}$ and $\mathbf{M}$ for a known measured leg forces by using equation (5). Two sample cases are as follows:

- For a combined (known) external load of $(0.7160.7160)^T$ N force and $(-7.8777.8770)^T$ N-mm, the measured values of force and moments are $(0.7350.7130.068)^T$ N and $(-9.6717.630-1.168)^T$ N-mm, respectively.
- For a combined (known) external load of $(0.9120.9120)^T$ N force and $(-10.03610.036-45.617)^T$ N-mm, the measured values of force and moments are $(0.9050.9260.146)^T$ N and $(-12.2098.999-45.957)^T$ N-mm, respectively.

The error in the measurement can be improved by more accurate application of external loads and by more accurate calibration.

## 5.2 Configuration 2

As mentioned earlier, the Configuration 2 with connection sequence $B_1 - P_2$, $B_2 - P_3$, $B_3 - P_4$, $B_4 - P_5$, $B_5 - P_6$, $B_6 - P_1$ was found to have singular directions for the three components of the moments. As in the Configuration 1, to arrive at the nominal dimensions of a six axis force-torque Gough-Stewart plat-

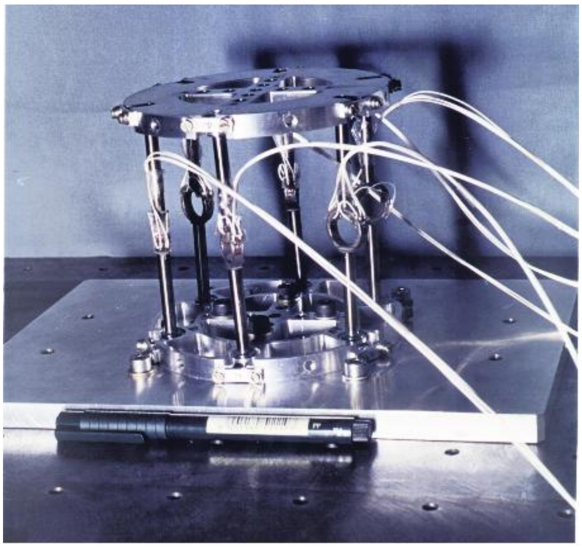

Figure 6: Force-torque sensor based on Gough-Stewart platform – Configuration 1(from [10])

Table 2: Nominal geometry of Configuration 2 sensor

| Base coordinates | | | Platform coordinates | | |
|---|---|---|---|---|---|
| Point No. | $x$ mm | $y$ mm | Point No | $X$ mm | $Y$ mm |
| $B_1$ | 21.65 | 12.5 | $P_1$ | 22.27 | 11.35 |
| $B_2$ | 21.65 | -12.5 | $P_2$ | 22.27 | -11.35 |
| $B_3$ | 0 | -25.0 | $P_3$ | -1.31 | -24.97 |
| $B_4$ | -21.65 | -12.5 | $P_4$ | -20.97 | -13.62 |
| $B_5$ | -21.65 | 12.5 | $P_5$ | -20.97 | 13.62 |
| $B_6$ | 0 | 25.0 | $P_6$ | -1.31 | -24.97 |

form based sensor sensitive to $M_x$, $M_y$ and $M_z$, extensive numerical simulations and additional requirements of maximum size and load carrying capability were taken into account. The nominal dimension for a near-singular configuration are as given in Table 2. It may be noted that the centre of the top platform is at a height of 37 mm from the centre of the bottom platform. The condition number of $[H]$ is about 707 which gives rise to fairly good mechanical amplification.

Due to the large load handling requirements, the flexure joint in a leg of this sensor is redesigned to make it stronger. This is shown in figure 7.

Figure 7: A typical leg of the sensor for Configuration 2 (from [25])

A CAD model of the sensor was made and as in the Configuration 1 sensor, extensive finite element analysis was performed to test the integrity of the sensor. The prototype sensor is shown in figure 8.

Figure 8: Force-torque sensor based on Gough-Stewart platform – Configuration 2(from [25])

Experimental tests are being conducted to obtain the performance and accuracy of the sensor in measuring all six components of force and moment. Preliminary tests show that the sensor is, as expected, sensitive to all the three components of moments and the sensor is quite accurate. A sample plot of the applied moment, $M_y$, versus the strain measured in leg 1 is shown in figure 9. As it can be seen that the deviation from a linear fit is quite small. More experiments are being planned to collect enough data so that the $[H]$ matrix can be estimated accurately and the sensor is made capable of actual measurements of all the six components of applied force and torque.

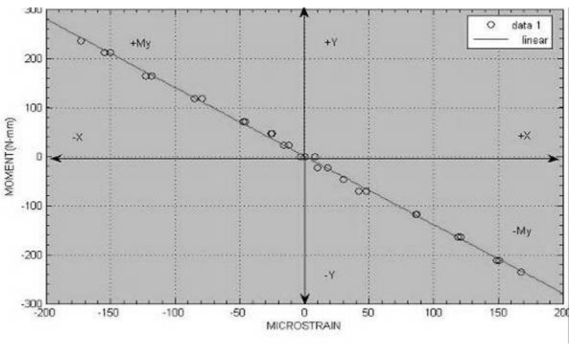


Figure 9: Applied moment versus strain in leg 1

# 6 Conclusions

This paper deals with the use of Gough-Stewart platform configuration for six component force-torque sensors. The Gough-Stewart platform based sensors can be either in isotropic or near-singular configurations, each with its advantages and disadvantages. In this paper, an analytic formulation to obtain the isotropic and singular configurations of $6-6$ Gough-Stewart platforms are presented. Two near-singular configuration six component force-torque sensors were designed, fabricated and tested. The experimental results demonstrate that Gough-Stewart platform based force-torque sensors, especially the ones in a near-singular configuration, have a lot of promise.

# Acknowledgment


The author wishes to thank his ex-students Dr. Sandipan Bandyopadhyay and Dr. R. Ranganath whose research included the isotropy study and near-singular configuration sensors, respectively. The author also thanks project staff M. Prashant, S. Bhavikatti and Deepak C. for help in design, fabrication and testing of the two sensors. The sensor developments were funded partly by grants from the Indian Space Research Organization through the Space Technology Cell at IISc, and their support is gratefully acknowledged.


# References


[1] V. E. Gough, "Contribution to discussion of papers on research in automobile stability, control and tyre performance", *Proc. of Auto Division, Inst. Mech. Eng.*, 1956-57.

[2] D. Stewart, "A platform with six degrees of freedom", *Proc. Inst. Mech. Eng.*, **180**(1), pp. 371-386, 1965.

[3] B. Dasgupta and T. S. Mruthyunjaya, "The Stewart platform manipulator: A review", *Mechanism and Machine Theory*, **35**, pp. 15-40, 2000.

[4] J.-P. Merlet, *Parallel Robots*, Kluwer Academic Press, Dordrecht, 2001.

[5] D. R. Kerr, "Analysis, properties and design of a Stewart platform transducer", *Trans. ASME, Jou. Mechanisms, Transmission and Automation in Design*, **111**, pp. 25-28, 1989.

[6] M. Sorli and N. Zhmud, "Investigation of force and moment measurement system for a robotic assembly hand", *Sensors and Actuators – A*, **37-38**, pp. 651-657, 1993.

[7] W. Hongrui, G. Feng and Huang Zhen, "Design of a six axis force/torque sensor based Stewart platform related to isotropy", *Chinese Journal of Mechanical Engineering*, Vol. 3, 1998.

[8] T. A. Dwarakanath, B. Dasgupta and T. S. Mruthyunjaya, "Design and developement of a Stewart platform-based force-torque sensor", *Mechatronics*, **11**, pp. 793-809, 2001.

[9] Z. L. Jin, F. Gao and X. H. Zhang, "Design and analysis of a novel isotropic six-component force/torque sensor", *Sensors and Actuators – A*, **109**, pp. 17-20, 2003.

[10] R. Ranganath, P. S. Nair, T. S. Mruthyunjaya and A. Ghosal, "A force-torque sensor based on a Stewart platform in a near-singular configuration", *Mechanism and Machine Theory*, **39**(9), pp. 971-998, 2004.

[11] K. Abe, T. Miwa, and M. Uchiyama, "Development of a 3 axis planar force/torque sensor for very small force/torque measurement", *JSME International Journal*, Series C, **12**(2), pp. 376-382, 1999.

[12] J. M. Paros and L. Weisboard, "How to design flexure hinges", *Machine Design*, pp. 151-156, 1965.

[13] S. Zhang and E. D. Fasse, "A finite-element-based method to determine the spatial stiffness properties of a notch hinge", *Trans. of ASME, Jou. of Mechanical Design*, **123**, pp. 141-147, 2001.

[14] P. J. Champagne, S. A. Cordova, M. S. Jacoby and K. R. Lorell, "Development of a precision six axis laboratory dynamometer", *NASA-CP-3147, 26th Aerospace Mechanisms Symposium*, pp. 331-348, 1992.

[15] D. M. Gorinevsky, A. M. Formalsky and A. Y. Schneider, *Force Control of Robotics Systems*, CRC Press, New York, 1997.

[16] M. Raghavan, "The Stewart platform of general geometry has 40 configurations", *Trans. of ASME, Jou. of Mechanical Design*, **115**, pp. 277-282, 1993.

[17] A. Ghosal, *Robotics: Fundamental Concepts and Analysis*, Oxford University Press, New Delhi, 2006.

[18] K. Y. Tsai and K. D. Huang, "The design of isotropic 6-DOF parallel manipulators using isotropy generators", *Mechanism and Machine Theory*, **38**, pp. 1199-1214, 2003.

[19] K. E. Zanganeh and J. Angeles, "Kinematic isotropy and optimum design of parallel manipulators", *International Journal of Robotics Research*, **16**, pp. 185-197, 1997.

[20] A. Fattah and A. M. H. Ghasemi, "Isotropic design of spatial parallel manipulators", *International Journal of Robotics Research*, **21**, pp. 811-824, 2002.

[21] Y. X. Su, B. Y. Duan and C. H. Zheng, "Genetic design of kinematically optimal fine tuning Stewart platform for large spherical telescope", *Mechatronics*, **11**, pp. 821-835, 2001.

[22] S. Bandyopadyay and A. Ghosal, "An algebraic formulation of static isotropy and design of statically isotropic $6-6$ Stewart platform manipulators", *Mechanism and Machine Theory*, **44**, pp. 1360-1370, 2009.

[23] R. A. Horn and C. A. Johnson, *Matrix Analysis*, John Wiley & and Sons, New York, 1975.

[24] A. Ghosal and B. Ravani, "A differential-geometric analysis of singularities of point trajectories of serial and parallel manipulators", *Trans. of ASME, Journal of Mechanical Design*, **123**, pp. 80-89, 2001.

[25] M. Prashant, S. Bhavikatti, R. Ranganath and A. Ghosal, "Analysis and design of a moment sensitive flexure jointed Stewart platform based force-torque sensor in a near singular configuration", *Proc. of ARMS-2005*, Bangalore, 2005.

[26] *Matlab Users' Manual (1994)*, The Mathworks Inc., USA

[27] *NISA II/DISPLAY III Users Manual*, Engineering Mechanics Research Corporation, USA, Version 7, 1997.