Studies on wobble mode stability of a three wheeled vehicle

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Abstract

A wobble instability is one of the major problems of a three wheeled vehicle commonly used in India, and these instabilities are of great interest to industry and academia. In this paper, we have studied this instability using a multi-body dynamic (MBD) model and with experiments conducted on a prototype three wheeled vehicle (TWV) on a test track. The MBD model of a three wheeled vehicle is developed using the commercial software ADAMS-CAR. In an initial model, all components including main structures like frame, steering column and rear forks are assumed to be rigid bodies. A linear eigenvalue analysis, carried out at different speeds, reveals a mode that has a predominantly steering oscillation, also called a Wobble mode, with frequency around 5 to 6 Hz. The analysis result shows that the damping of this mode is low but positive up to the maximum speed of the TWV. However the experimental study shows that the mode is unstable at speeds below 8.33 m/s. To predict and study this instability in detail, a more refined model of the TWV, with flexibility in three important bodies, was constructed in ADAMS-CAR. With flexible bodies, three modes of a steering oscillation were observed. Two of them are well damped and the other is lightly damped with negative damping at lower speeds. Simulation results with flexibility incorporated shows a good match with the instability observed in experimental studies. Further, we investigated the effect of each flexible body and found that the steering column flexibility is the major contributor for wobble instability and is similar to the wheel shimmy problem in aircraft.

Key words: Three wheeled vehicle, steering oscillation, Wobble, shimmy, structural flexibility, stability.

1. Introduction

The three wheeled vehicle is a very common public transport vehicle in India, with a maximum speed of about 14 m/s. This vehicle, commonly known as an Auto-rickshaw is shown in Figure 1a. Similar vehicles are used throughout the world, especially in Asian countries, for public transport as well as to carry freight. The total weight of the vehicle is around 650 kg including the driver and three passengers. It has one front wheel with linkage (trailing or leading) suspension attached to the steering column and two rear wheels attached to corresponding swinging arms that are pivoted to the frame. The steering system construction and wheel sizes are similar to a scooter, and the mechanical trail is much less compared to the motorcycle described in [1]. The dynamics of this vehicle is peculiar and very limited research is available in published form. Few studies are published [2 -- 5] for ride and handling characteristics on bumpy roads using rigid body models with suspension not included. A significant amount of research has been published [6 -- 11] on tilting three wheeled vehicles. The dimensions, mass, geometrical parameters and kinematics of the vehicles that are studied in these references are quite different from the kind of vehicle that is considered in this paper.

A few papers are available for the kind of three wheeled vehicle that is considered in this paper. Ramji and his co-authors [12] studied the effect of suspension stiffness and damping on vertical acceleration of the sprung mass using an analytical model with rigid bodies. Gawade and his co-authors [13] developed a six degree of freedom analytical model considering the effect of suspension and tire slip. Reference [14], that shows the importance and severity of wobble instability of the three wheeled vehicle, is one of the papers relevant to the current study. This work reports an experimental study to understand the effect of preloads of a dry friction damper on wobble instability. However, the reason for this instability is not investigated and this is one of the motivations for the current study.

We mention that there have been prior studies on the effect of structural flexibility on weave and wobble stability for two wheeled vehicles [15--17]. These models are with simpler representation of flexibility and results of these studies showed that the structural flexibility has influence on stability of a two wheeler in specific speed zones. However, this kind of simple representation of flexibility may not have the fidelity to solve some of the problems, perhaps like the problem that is studied in this paper. In the current paper, we offer a detailed study of wobble instability problem of a three wheeled vehicle. The main focus is to model the vehicle close to reality to predict the frequency and damping of the wobble mode more accurately. The implication of this study is that the flexibility of the main structures shall be included in the dynamic model of a three wheeled vehicle; such considerations will predict the stability issues more precisely in design stage. To predict the wobble instability and study it in detail, we have used finite element based flexible model. Although flexible models based on finite element meshes is common in multi body simulation, its use in study of stability for two and three-wheeled vehicles has not been published more often.

Now we present the brief description of the study. Initially, we study the wobble mode using the model with rigid bodies that has 25 degree of freedom. The detailed description of the model and other simulation details is given in [18] for saving space in this paper. However a brief description of the model is presented in the next section for the completeness of the paper. Using this multi-body dynamic model, linear eigenvalue analysis is carried out to study the wobble stability of the vehicle. It was observed that one mode that involves predominantly steering oscillation and less body movements has very low damping. This mode is similar to the wobble mode of a two-wheeler; hence we call this mode Wobble.

Further, experiments are conducted on a test track with a prototype vehicle for different vehicle speeds. An important note is that, most of the vehicles that are in production have a friction damper (see [14] for more details) at steering to arrest these oscillations. In this study the experiments are conducted without the friction damper. The vehicle speed, steering torque and steering angles are the parameters measured during the experiment. The wobble is unstable at lower speeds (below 8.33 m/sec) and marginally stable at higher speeds. This is an important finding from the experimental result.

The frequency of the wobble mode calculated from the simulation result match reasonably well with the experimental result. However, the simulation result shows that the damping of the mode is positive which does not agree with the experimental result. Overall, the simulation with rigid body model does not predict the wobble instability that is observed in experimental study.

Hence, structural flexibility was incorporated in the frame, steering column and trailing arms (rear forks or rocking arms). The linear eigenvalue analysis with flexible bodies shows that there are three steering oscillation modes that have the frequency around 5 Hz. Two extra modes were observed along with a mode similar to the mode observed in rigid body model. The first mode consists of predominantly roll motion apart from steering oscillation. The damping of this mode is positive and much higher compared to other two modes. The second mode is similar to the mode observed in rigid body model. The damping of this mode is very less compared to the wobble mode that is observed with rigid body model. And the damping of this mode is negative up to some speed which agrees with the experimental result. It is one of the important finding of this study. The third mode has a constant frequency and damping throughout the speed. Hence it might be due to flexibility of the three parts.

Finally, simulations are carried out keeping one of the bodies as flexible and other two as a rigid. These set of simulations leads to another important finding of this study, namely that the flexibility of steering column is the reason for low and negative damping of wobble.

This paper is organized as follows. In Section 2 a brief description of the multi-body dynamic model of a three wheeled vehicle using ADAMS-CAR is presented. The model is used to carry out linear eigenvalue analysis to study the wobble instability of vehicle and is presented in Section 3. In Section 4, the details of experimental study with the prototype vehicle are presented and the experimental results are compared with simulation results. As mentioned before, the experimental study shows that the rigid body model does not capture fully the dynamic behaviour of steering oscillation (Wobble) mode. In order to improve the model, structural flexibilities are incorporated in the frame, steering column and trailing arm (rocking arm). The details of flexible body modelling and the study of the wobble instability using this model is described in Section 5. The results show a good match with experimental results. In Section 6, additional studies with flexible bodies are presented to understand the effect of each flexible body on the wobble instability and these parametric studies of payload, steering offset etc. are presented in Appendix A. The main conclusions of this study are presented in Section 7.

2. Model description

In this section we describe the development of a dynamic model of a three wheeled vehicle (TWV), using ADAMS-CAR. A schematic of the model is shown in Figures 1b and 1c. The rigid model has 25 degree of freedom. These are: 6 degree of freedom for the frame plus rigidly attached rider, 6 degree of freedom for the powertrain, 3 rotations of each trailing arm (total 6), 1 rotation of each rear suspension (total 2), 1 rotation at the steering pivot, 1 front trailing link rotation and 3 wheel rotations.

Among the above degrees of freedom, two out of three rotations in each trailing arm occur due to rubber bush, and are small; the main rotation is due to the rear suspension displacement. The translational degree of freedom for the trailing arm is removed using a bush and spherical joint to attach it to the frame. The rear suspension is attached to the trailing arm and frame thru rubber bushes and appropriate joints in a way that the relative rotation between the frame, suspension and trailing arm depends on rubber bush torsional stiffness. This results in one degree of freedom for each rear suspension. The power train is attached to the frame thru four bushes results in 6 degree of freedom. These are the extra degree of freedom (total 12: 2 rotations of each trailing arm, 1 rotation of each rear suspension and 6 degree freedom of power train) are added to consider the bushes in the interest of making the model close to reality. However, it is observed that the effect of these bushes on the results presented in this paper is very small. Hence, almost the same results can be obtained by using a simplest 13 degree of freedom model. Here, we will consider the rigid body model with 25 degree of freedom.

Three wheeler dimensions and inertia properties, as well as tire and suspension properties, can be incorporated into the ADAMS-CAR model. Front and rear suspensions are modelled using simple spring and dashpot elements. Some parameters describing the vehicle layout, as well as the mass and inertia properties of various subsystems of a typical three-wheeler are listed in Appendix B. The net wheel reaction forces for the stationary vehicle and the centre of gravity (CG) location match our prototype vehicle within the precision of experimental measurements. Finally, tire-ground interaction is modelled using local stiffness and slip relations (using the Magic formula [18, 19, and 20]). The control systems and the method of controlling the vehicle are as described in *eprintsrvr report* [18]. The details of multi-body dynamic model are described in the same *report*. However, for the sake of completeness, a brief description of the model is described below.

The model consists of eight different subsystems, which are defined and assembled together, to make the full vehicle assembly as shown in figure 1 c. The subsystems are: frame assembly, steering system assembly, front suspension assembly, trailing arm assembly (including shock absorbers), brake system, power train, front wheel and rear wheels.

The procedure to model a subsystem starts with template creation, communicators, hard points etc. and is described in detail in reference [18]. Since the model is very complex it is recommended to use the qualitative inferences of the study.

Figure 1a: Picture of a three wheeled vehicle

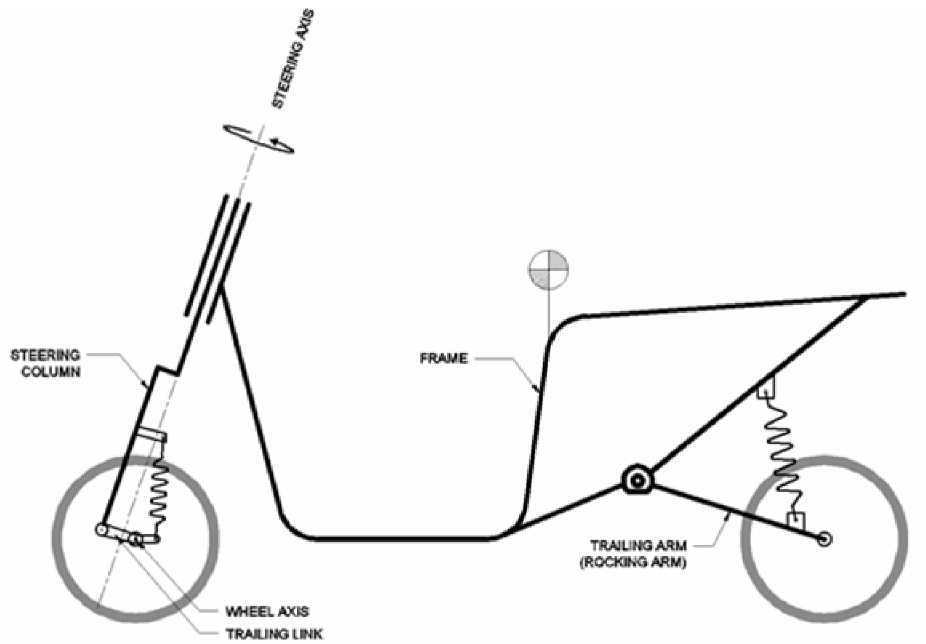

Figure 1b: A schematic of a three wheeled vehicle.

3. Study of steering oscillation (Wobble) with rigid body model

In this section, the linear eigenvalue analysis of the three wheeled vehicle travelling on a straight path is presented. This analysis is done using linearized quantities at an instant of interest and is conducted in two steps.

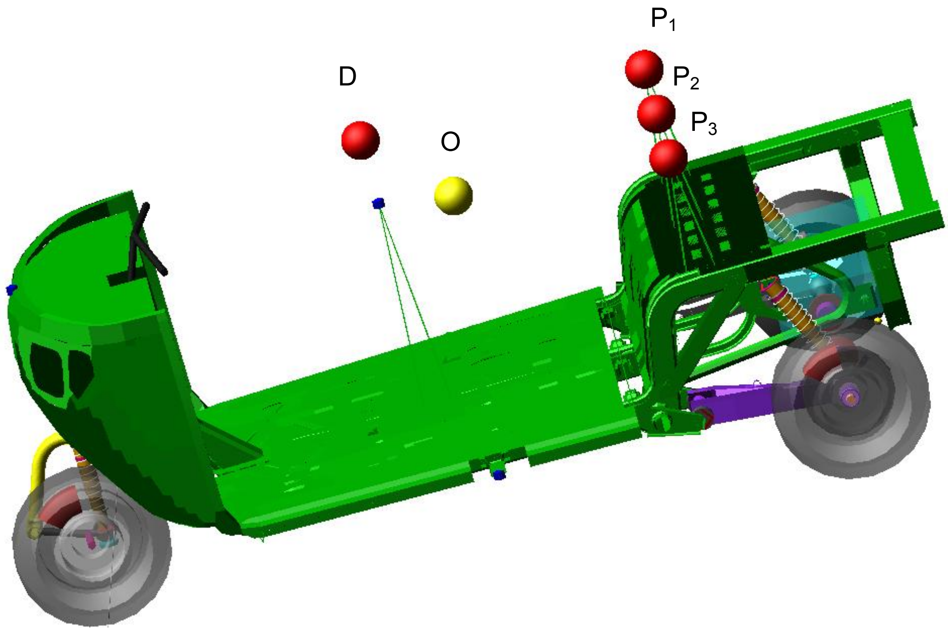

Figure 1c: Multi-body dynamic model of a three wheeler. In this figure D, P_1, P_2, P_3 and O represent the CG location of the driver, three passengers and the chassis.

In the first step the values are generated for a state that is then used for the linearized analysis in the second step. The state of interest here is forward motion at a steady speed. To attain this, the vehicle is allowed to achieve steady-state straight running. The output of the first step is saved for subsequent use. In the second step, 'Normal mode analysis' is chosen to find eigenvalues governing the linearized dynamics of the vehicle with the output data of the first step (See section 4.3 in [18] for a detailed description of the steps). Using the above two steps, the simulation is carried out for various speeds with two different choices of mechanical trail. It may be mentioned that these simulations are performed for the rigid body model and no structural flexibilities are included.

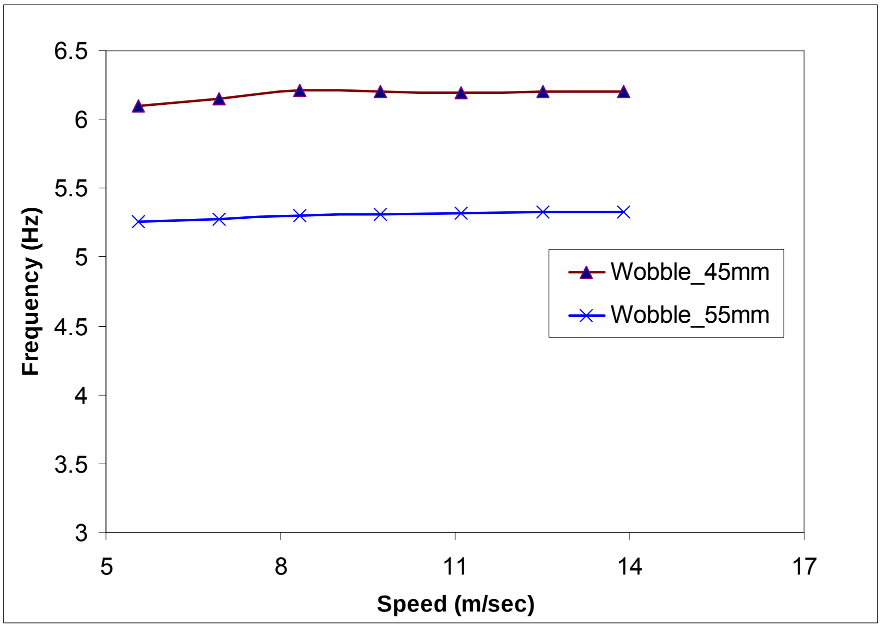

Figure 2a: Variation of Wobble mode frequency (for 45 mm and 55 mm steering offset) with speed.

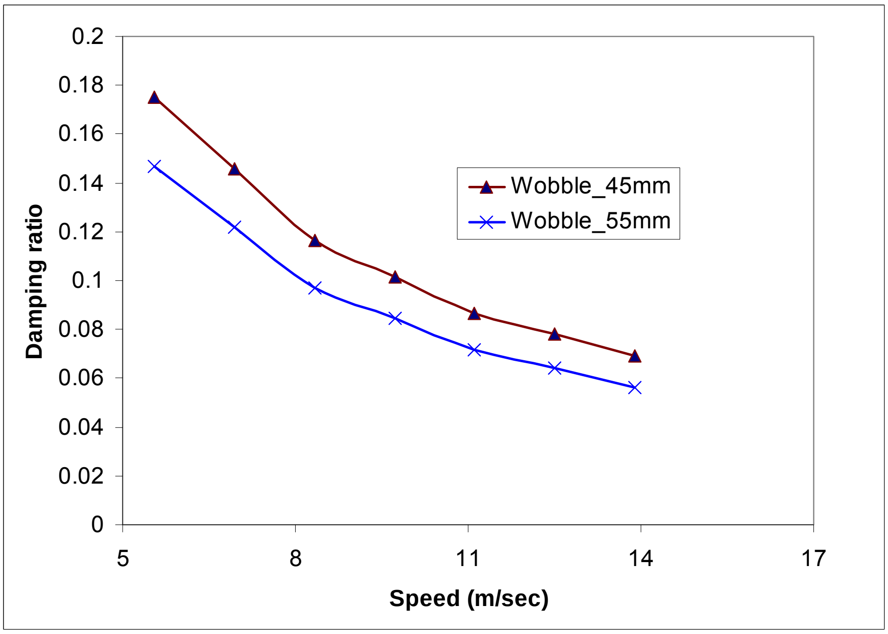

Figure 2b: Variation of Wobble mode damping ratio (for 45 mm and 55 mm steering offset) with speed.

The simulation results show that there are 50 eigenvalues including 6 rigid body modes. Each mode shape was examined visually and only the steering oscillation mode is selected for presentation here. This mode predominantly involves steering oscillation, small body movements, and has low damping. The mode seems similar to the Wobble mode of a two-wheeler and hence we have called it a Wobble mode.

Figure 2a shows the imaginary parts of these Wobble mode eigenvalues -- each divided by 2π to give frequencies in Hz. The variation of frequency with the vehicle speed is plotted for Wobble mode. The results show that the frequency of oscillation is nearly constant, around 5.5 Hz for 55 mm steering offset and around 6.1 Hz for 45 mm steering offset. Similarly, Figure 2b shows the variation of damping ratio with vehicle speed. The results show that the damping of the mode is positive at all speeds below 13.89 m/sec (50 kmph) and the damping decreases as the speed increases. Overall, the results show that the mode is stable. However, in the experimental study, a steering instability (wobble) has been observed especially at speeds below approximately 8.33 m/sec. The detail of this experimental study is described in the next section.

4. Experimental study of Wobble mode

In this section, the experimental results of steering oscillation measured on a flat test track are presented. Steering angle, steering torque and speed of the vehicle are measured. A steering potentiometer is used to measure steering angle and a torque sensor to measure steering torque. The weight and inertia of the torque sensor is very small compared to the steering system; hence, its effect on the dynamics is neglected. The vehicle speed is measured using a ‘V box’**,** which is a GPS based system.

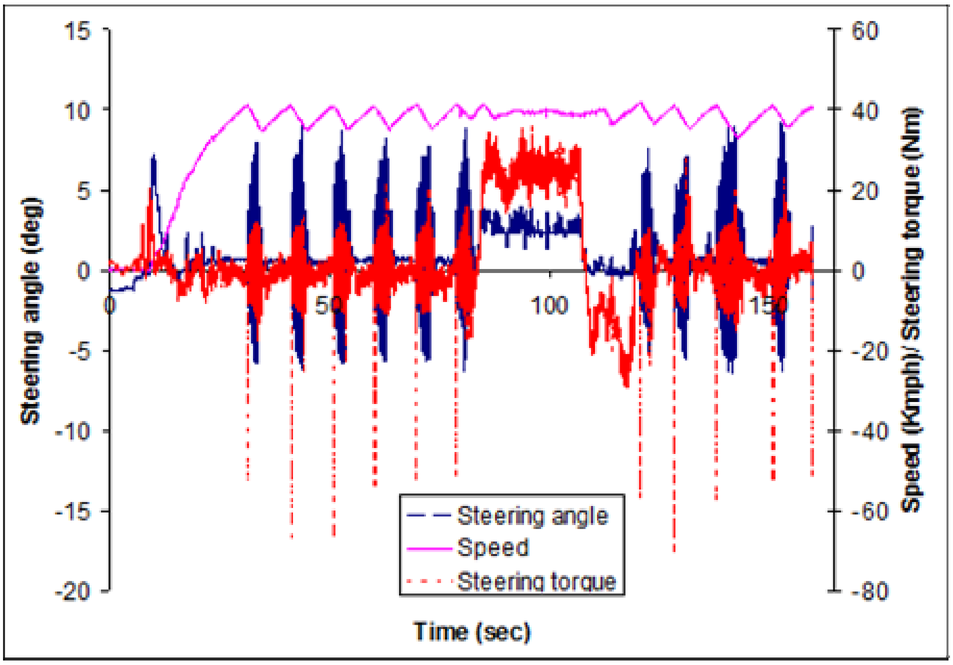

Figure 3: A typical result of experiment: measurement of speed, steering angle (degree) and steering torque (Nm).

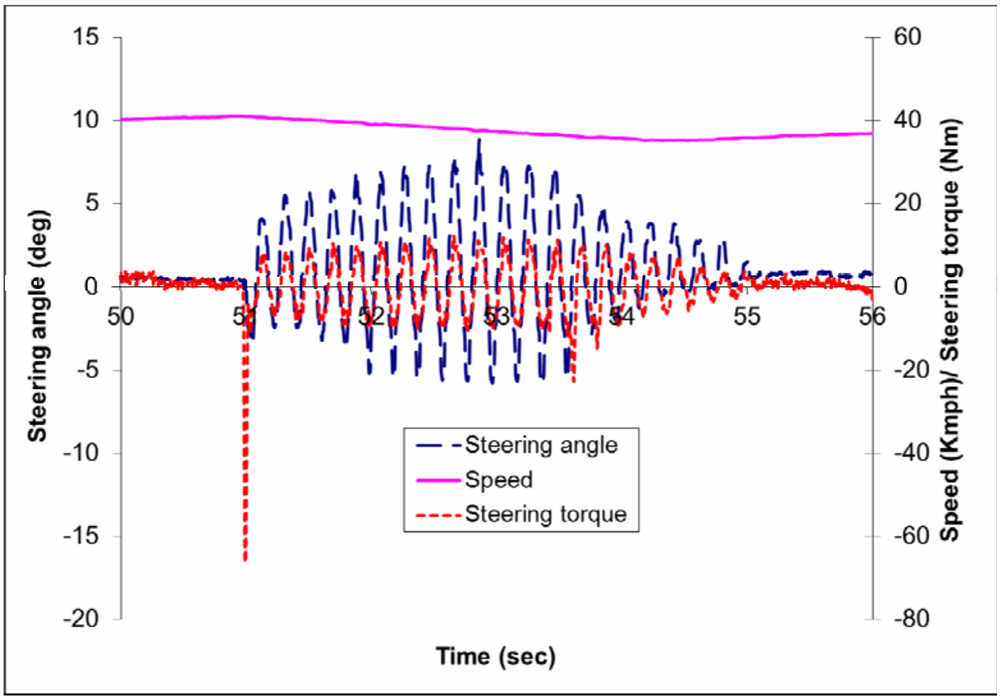

Figure 4: Steering angle and torque measurement at 11.11 m/sec (40 kmph), steering offset 45mm.

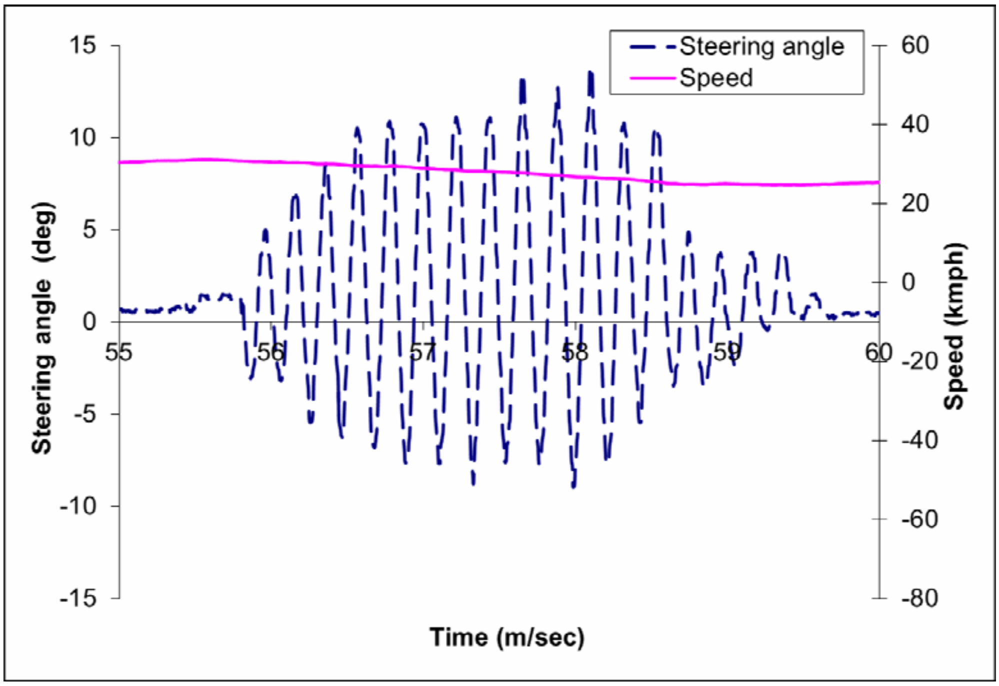

Figure 5: Steering angle measurement at 8.33 m/sec (30 kmph), steering offset 45mm.

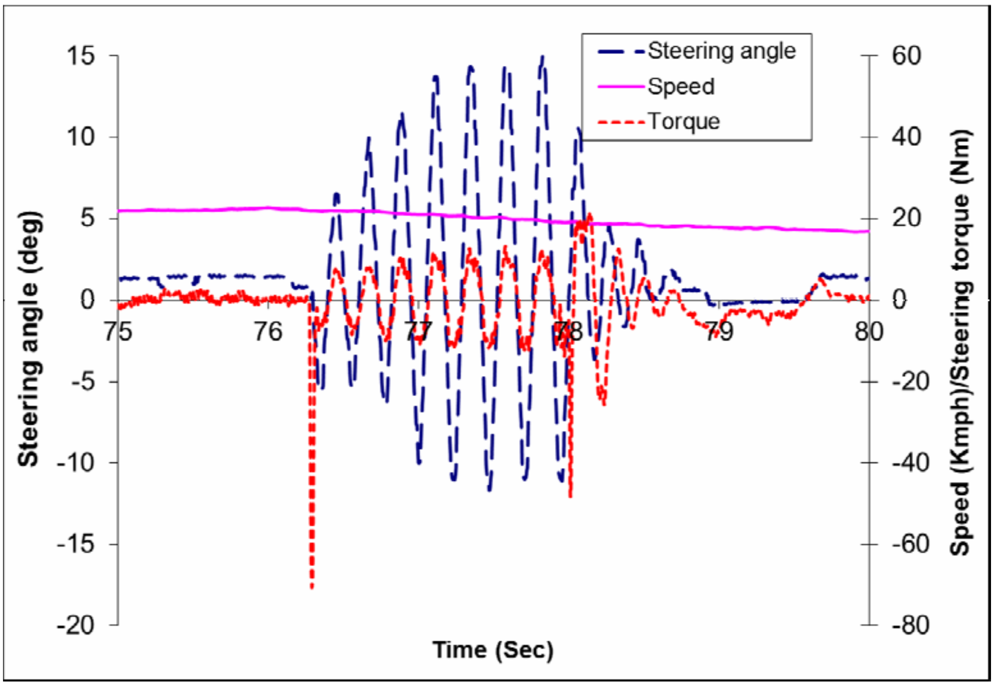

Figure 6: Steering angle and torque measurement at 5.56 m/sec (20 kmph), steering offset 45mm.

A typical signal of the test is shown in Figure 3. It consists of ten trials at the same speed of 11.11 m/s. The test is conducted as follows: the vehicle speed is maintained at 11.11 m/s (40 kmph) and a small impulse is given at handlebar to excite the steering oscillation mode. For example, as shown in Figure 4, the first impulse in steer torque curve is given just after 51th second. After the impulse is given, the vehicle is allowed to move without any control from the driver (hands off condition with no speed and directional control). In the same way, the test is repeated for speeds of 8.33 and 5.56 m/sec and the experimental results are shown in Figures 5 and 6, respectively. During the tests, there is a slight drift of the vehicle from straight path. Hence, after few oscillations, vehicle is controlled with steering torque input from the driver due to the limitation of the width of the track. In Figure 6 the torque input around 78th second is an example of this correction. Another point is that it is very difficult to maintain constant vehicle speed during testing. The low deceleration of the vehicle will increase the front wheel normal reaction which may cause minute drop in wobble damping. This drop in the wobble damping will be less compared to the order of problem under investigation in this paper. Hence, it can be assumed that the qualitative results and inferences of this study may not alter.

The Figures 4 -- 6 show the test results of prototype vehicle with a steering-offset of 45 mm for three different speeds. The result shows that the vehicle has very low damping and appears to be unstable for all the speeds with steering oscillations of 15 to 30 degrees. The results of the steering angle show that the damping is positive above 8.33 m/sec and is negative at 5.56 m/sec. Further, it is observed that the vehicle is unstable below 8.33 m/sec. Also, the experimental result shows that the frequency of steering oscillation increases with speed as shown in table 1 which does not agree with simulation result presented in previous section. However the frequency of steering oscillation matches reasonably well with the simulation result.

Overall, the Wobble frequency found using the rigid multi-body model shows reasonably good match with the experimental result. However the simulation result showed that the damping of the mode is always positive, which is not matching with the experimental result. The reason for the instability is not clear with the rigid body model and this lead to the incorporation of flexible bodies into the multi-body dynamic model. The detail of this investigation is reported below.

Vehicle Speed (m/sec)	2.78	5.56	8.33	11.11
Frequency (Hz)	4.01	4.11	4.58	5.13

Table 1: Variation of the frequency of steering oscillation with vehicle speed – experimental result.

5. Incorporation of flexibility

As mentioned earlier, to capture the Wobble instability, the frame, trailing arm and steering column are modelled as flexible bodies. It is not very easy to accurately model structural flexibility in these components and only the major structure of each part that contributes towards stiffness is taken into account. The added flexibility is shown schematically in the Figures 7 -- 9. For the frame, 125 natural modes with the maximum frequency of 1998.7 Hz are considered. Similarly, for the steering column, 13 modes are considered with the maximum frequency of 1653.22 Hz and for the trailing arm, 20 modes have been considered with the maximum frequency of

1976.42 Hz. The detailed description of flexible bodies, mode shapes and its frequencies are mentioned in the *eprintsrvr* report [18].

The three rigid bodies are replaced with the flexible bodies and all other vehicle parameters kept same as rigid body model. The simulations are carried out to find eigenvalues at various speeds. Out of many modes, three steering oscillation modes were observed unlike in the case of rigid body model in which only one mode is observed. The variation of the frequency and damping of these three modes is shown in Figures 10a and 10b. The mode shapes of these three are observed to be different and are described below. The mode that has steering oscillation and roll motion is denoted as Wobble 1. For the second one, the mode shape is similar to the mode observed in rigid model (predominantly steering oscillation, less roll and yaw motions) and is called Wobble 2. The third mode consists of predominantly steering oscillation along with structural flexures of steering column and head pipe of the frame, and is denoted Wobble 3. Out of the three, Wobble 1 is of lower frequency and its frequency decreases with speed. It is well dampened and hence it is a stable mode. The Wobble 3 shows a constant frequency and damping over the speed range, which is expected, since it is predominantly due to structural flexibility. The Wobble 2 is similar to the mode observed in the rigid body simulation and it showed an increase of frequency with speed. Its value, however, is a little lower compared to rigid body model for lower speeds up to 11 m/sec and it matches with the frequency of rigid body model at higher speeds. The most important result is that the damping of the Wobble 2 mode increases with the speed unlike in the case of rigid model. The damping of the Wobble 2 mode is quite low and it showed a negative damping up to the speed of 9.0 m/sec.

The frequency and damping ratio of wobble obtained from both simulation and testing is compared in Figures 11a and 11b. In experimental study, the frequency is measured using FFT of the signal and damping is measured using logarithmic decrement method. In the figures, the simulation result with the rigid body model is denoted as ‘Wobble_rigid’, the simulation result (Wobble 2) with the flexible model is denoted as ‘Wobble_flexible’ and the experimental result is denoted as ‘Wobble_experimental’. The experimental result showed that the Wobble mode is unstable at a speed of around 5.5 m/sec and had almost zero damping from 8.3 m/sec onwards. Further it indicates that the mode may show very low positive damping at higher speeds. The damping ratio of simulation result with rigid body model is positive and significantly high compared to the results of both experimental and simulation with flexible model. However, there is a reasonably good match between the experimental results and the simulation results with the flexible model. Overall, these findings shows that the modelling of structural flexibility is very important to study the three-wheeler wobble instability.

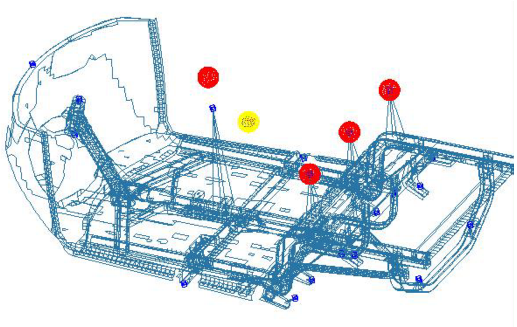

Figure 7: Flexible model of the frame.

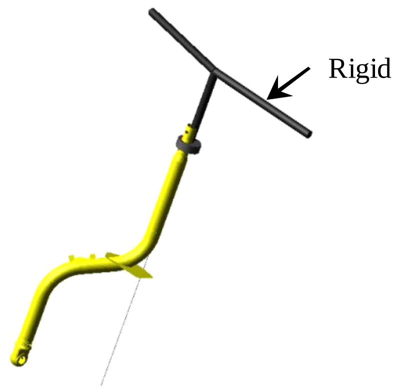

Figure 8: Schematic of flexible model of steering column.

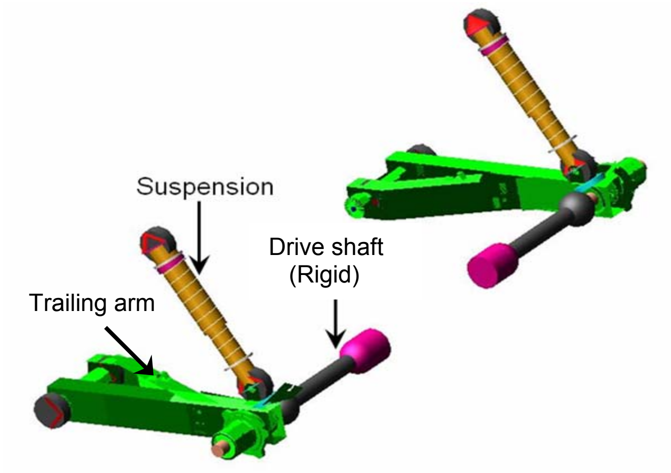

Figure 9: Schematic of rear suspension system with flexible model of trailing arm

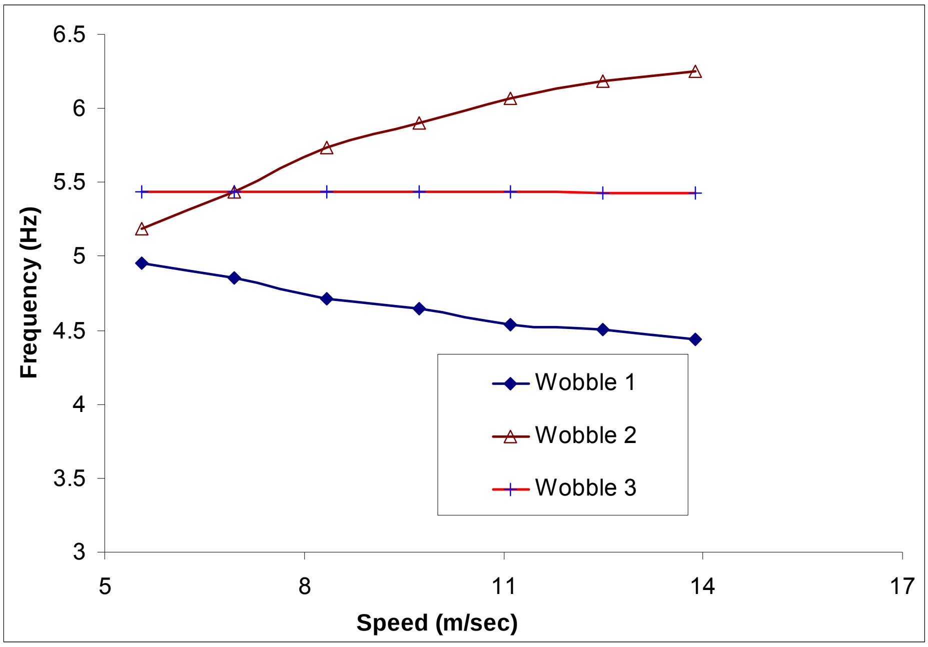

Figure 10a: Variation of Wobble mode frequency (for 45 mm steering offset) with speed for flexible chassis.

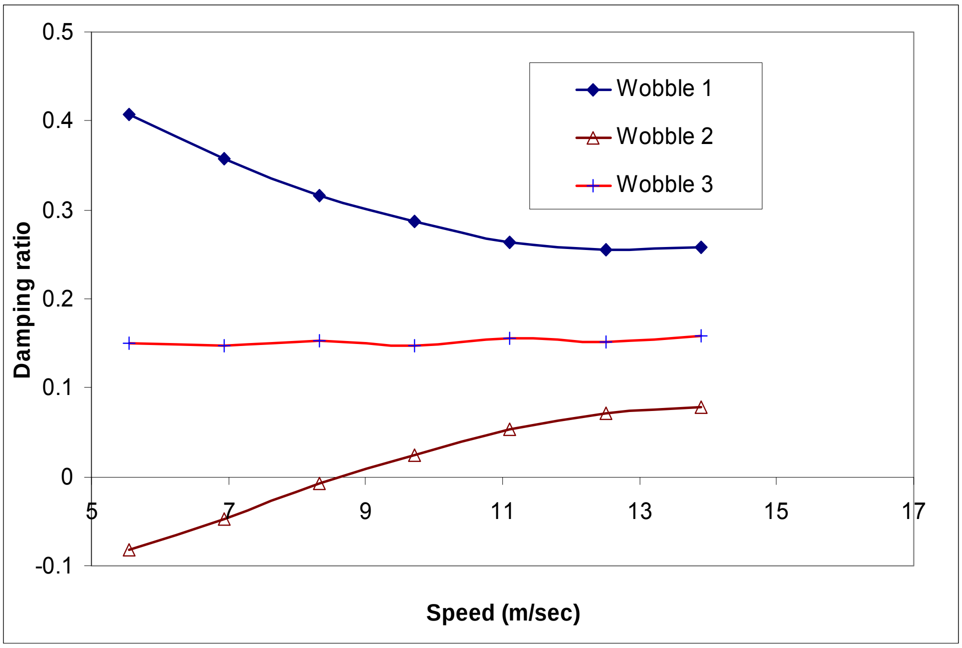

Figure 10b: Variation of Wobble mode damping (for 45 mm steering offset) with speed for flexible chassis.

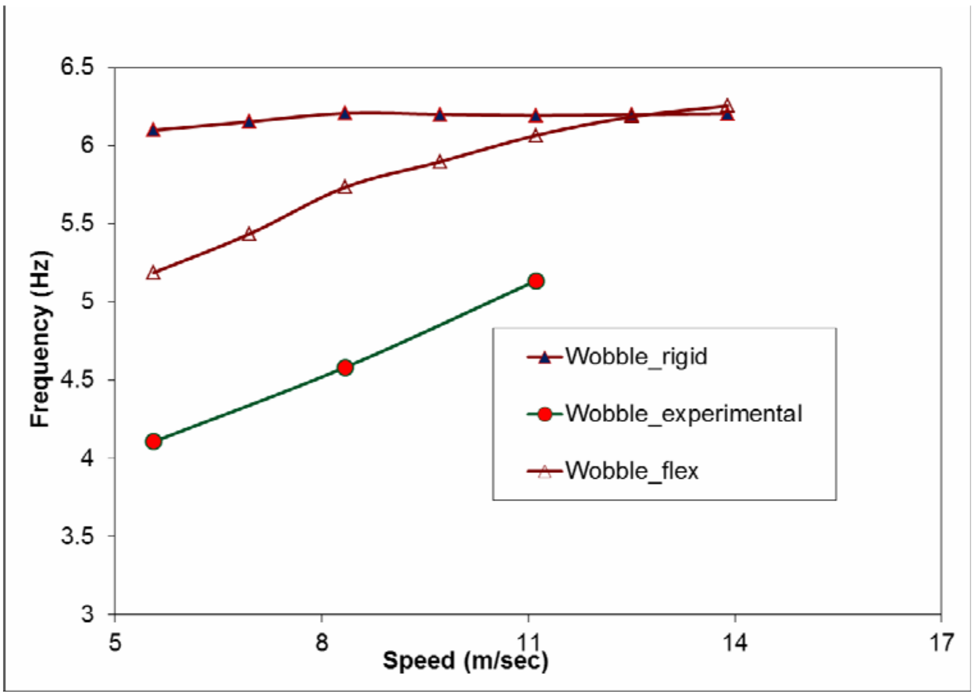

Figure 11a: Variation of Wobble mode frequency (for 45 mm steering offset) with speed.

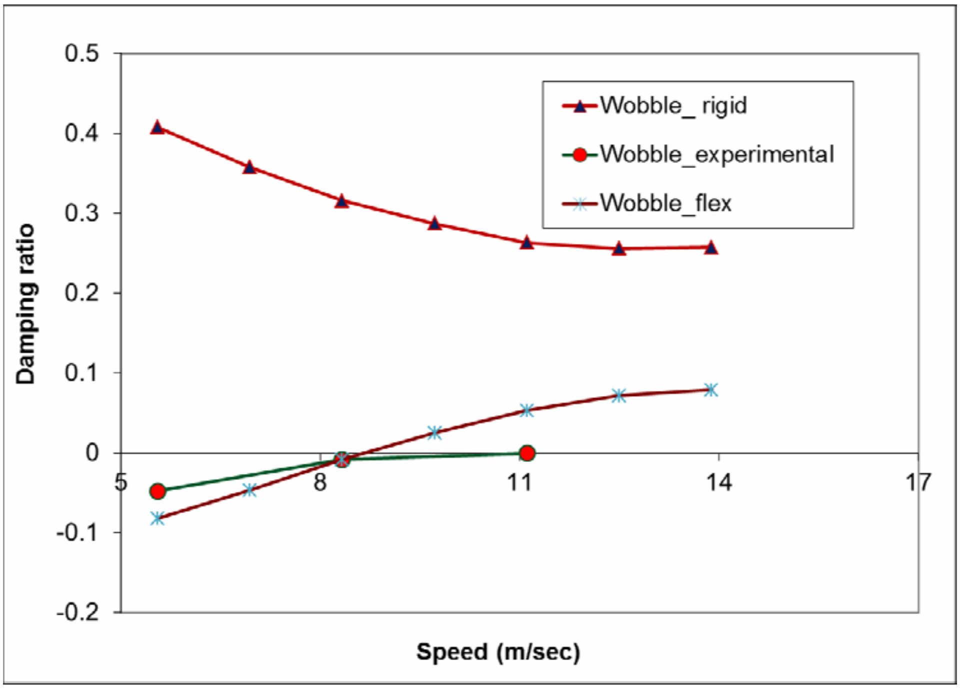

Figure 11b: Variation of Wobble mode damping (for 45 mm steering offset) with speed.

Further, these modes were studied with different mechanical trail, and the results are presented in the figures 14 – 17, Appendix A. An important finding of this study is that the decrease in mechanical trail (increase in steering offset) changes the modal interaction of Wobble 1 and Wobble 3 to occur at higher speed. At higher speeds, the damping of Wobble 2 is better with smaller trail; hence instability may not occur.

6. Identification of critical structure for Wobble stability

As discussed above**,** the modelling of structural flexibility captures the Wobble instability that is observed experimentally. A more detailed study was conducted to identify the influence of flexibility of each structure. Simulations are conducted with one part as rigid and other two as flexible and Figures 12 and 13 are for Wobble 2. Two more simulations are carried out and compared with rigid body model (labelled as ‘All rigid’) and flexible model (labelled as ‘Flexible chassis’). The two simulations are: 1.) The frame is modelled as rigid keeping steering column and the trailing arm as flexible labelled as 'frame rigid' in the figures, 2.) The steering column is modelled as rigid keeping frame and trailing arm as flexible labelled as 'steering column rigid' in the figures. The result shows that the wobble frequency of rigid body model is higher compared to the flexible model especially at low speeds (speed below 12.5 m/sec). The drop in the frequency with flexible model at low speeds is mainly due to frame flexibility. The simulation result (figure 13) also show that steering column flexibility is very important parameter to control the Wobble damping ratio compared to the other two flexibilities. Further, it shows that the steering column flexibility may be one of the main contributors for low damping as observed in experimental study. This is the second important finding of this study.

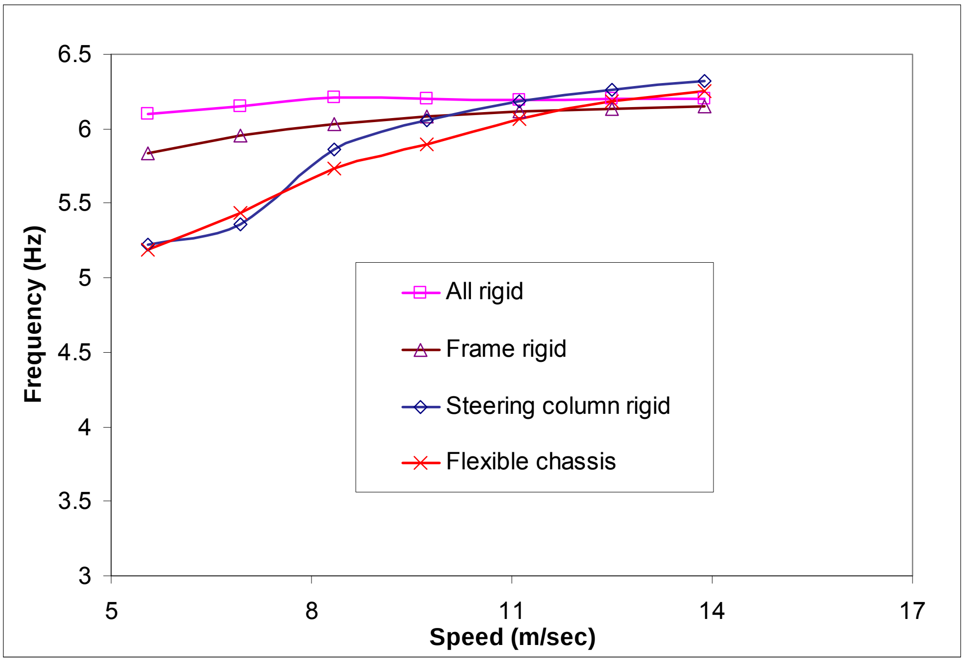

Figure 12: Variation of Wobble mode frequency (for 45 mm steering offset) with speed by changing rigid to flex and vice versa.

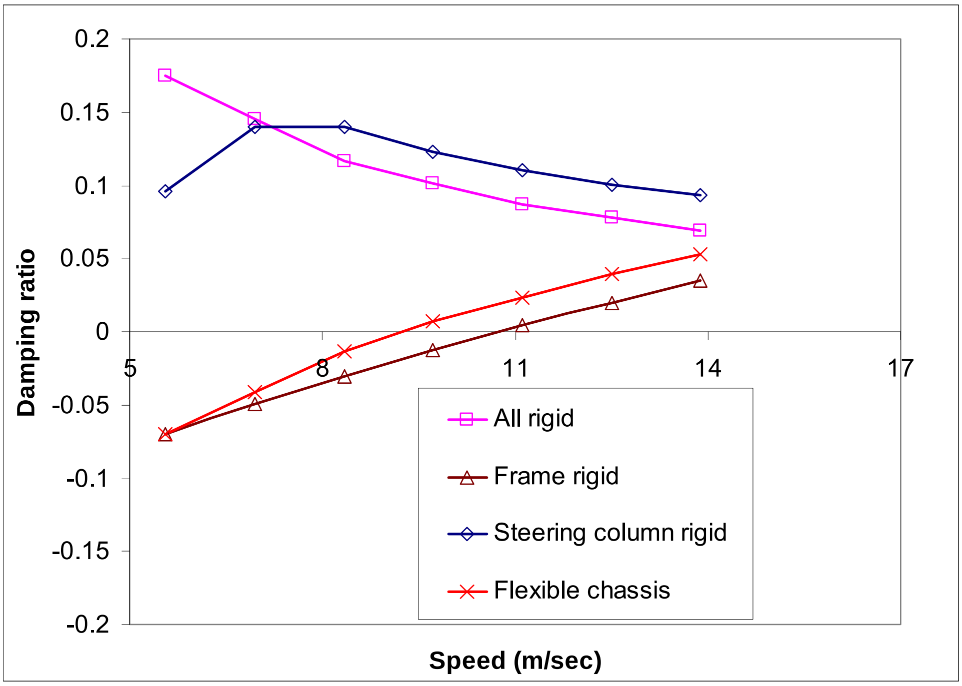

Figure 13: Variation of Wobble mode damping (for 45 mm steering offset) with speed by changing rigid to flex and vice versa.

7. Conclusions and future work

A detailed simulation of a three wheeled vehicle using a multi body dynamic model is very useful in industry, especially in the design stage. One important aspect of this vehicle is instabilities of steering oscillations especially at lower speeds. Simple models using rigid bodies do not capture this instability and a multi-body dynamic model, including flexibility, is developed and validated using the experimental results. Using this model it was found that structural flexibility modelling is essential to capture the wobble mode instability that is observed experimentally. Also, it was found that steering column flexibility may be one of the main reasons for these instabilities. The dynamic model, results and findings of this study can be used not only in future industrial design oriented studies, but also will lead to improved understanding of three wheeler dynamics as well, especially the wobble instability. Future work will include experimental study of wobble with steering column flexibility and study of modal interactions of various modes that may lead to instabilities at particular vehicle speeds.

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Appendix A

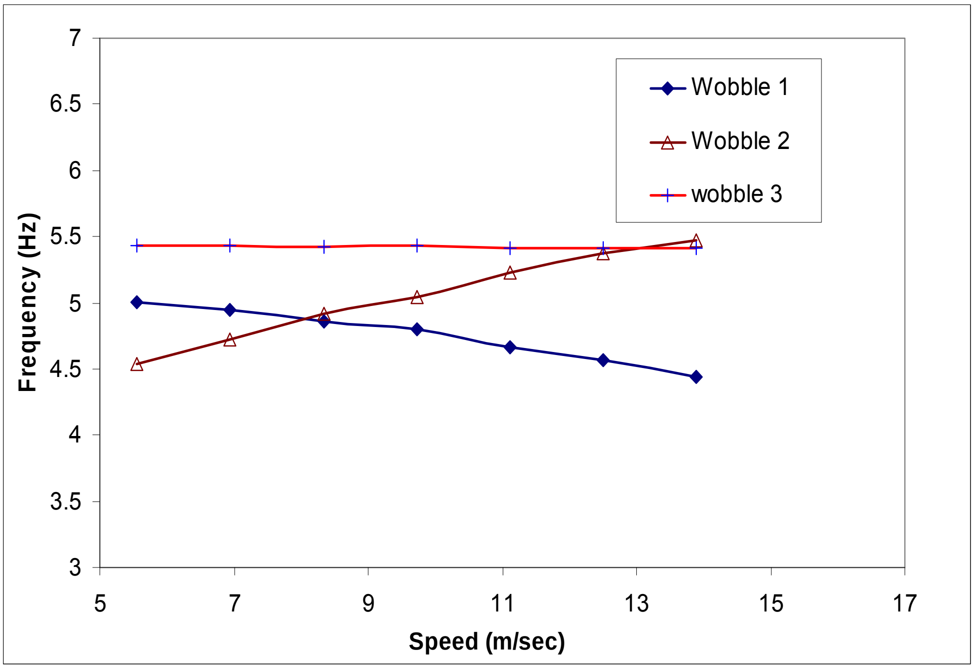

Figure 14: Variation of Wobble mode frequency (for 55 mm steering offset) with speed for flexible chassis.

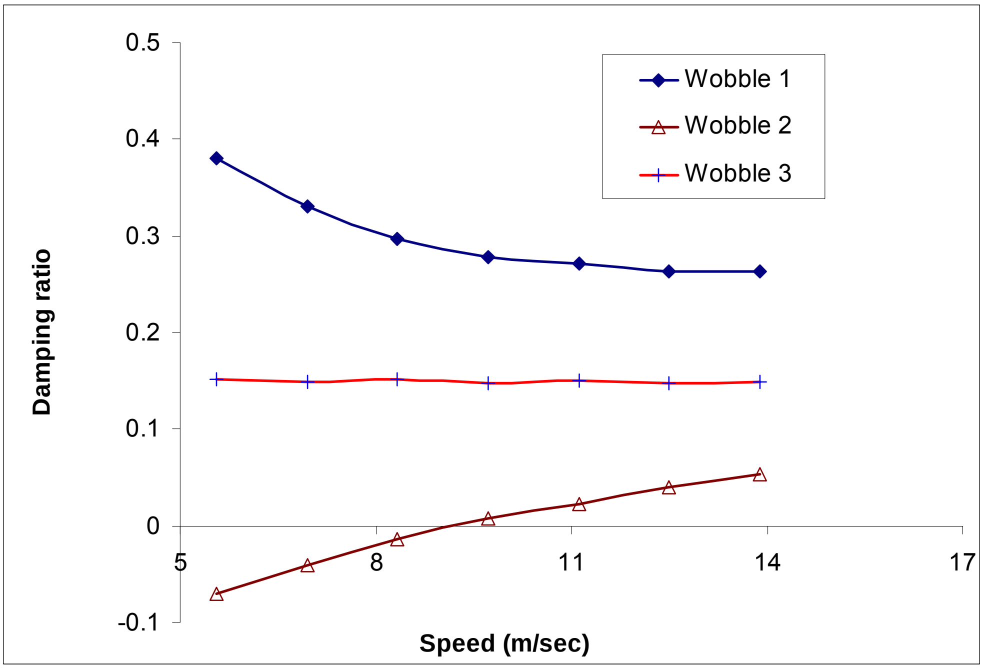

Figure 15: Variation of Wobble mode damping (for 55 mm steering offset) with speed for flexible chassis.

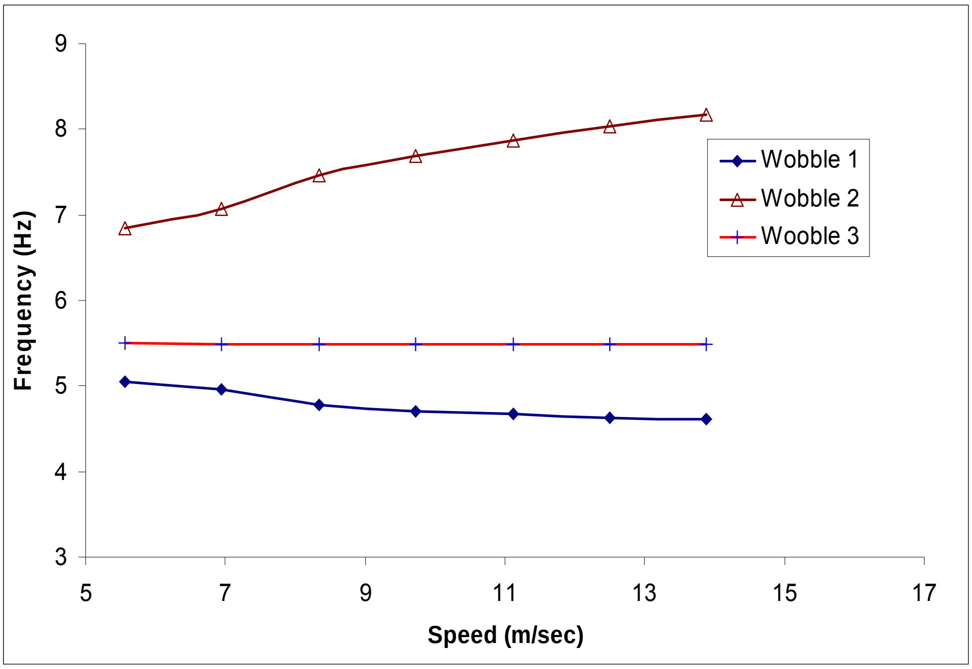

Figure 16: Variation of Wobble mode frequency (for 20 mm steering offset) with speed for flexible chassis.

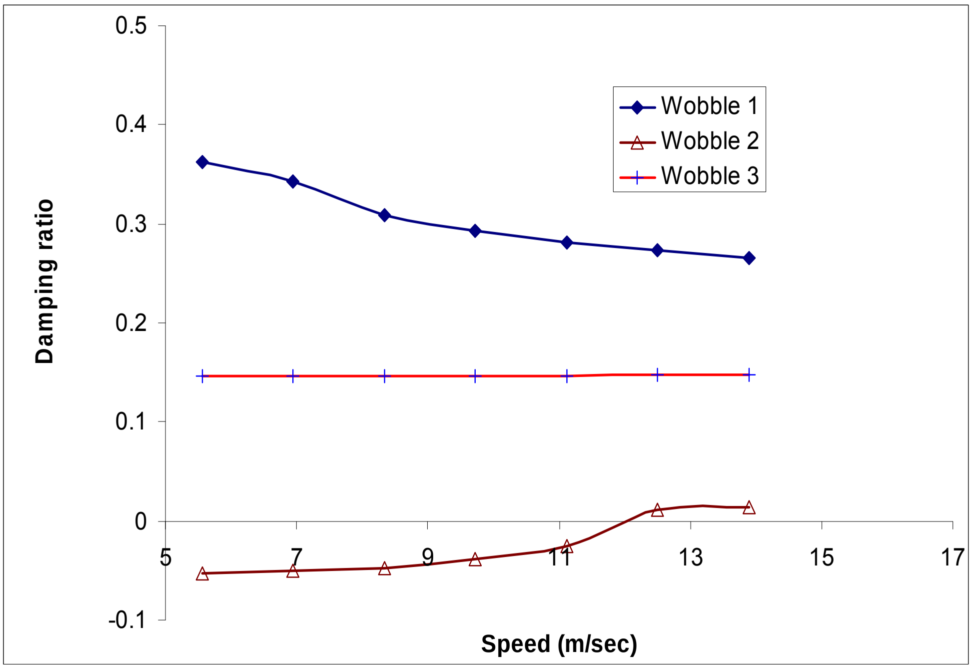

Figure 17: Variation of Wobble mode damping (for 20mm steering offset) with speed for flexible chassis.

Appendix B

S.no	Vehicle parameter	Value
1	Wheel base	1980 mm
2	Wheel track	1150 mm
3	Trailing link length and angle	168 mm & 15 deg
4	Caster angle	19 deg
5	Steering offset	55 mm
6	Trailing arm length & angle	395 mm & 5.8 deg.
7	Front tire size (Radius × width)	203 mm × 101 mm
8	Rear wheel size (Radius × width)	203 mm × 101 mm

Table 2: *Parameters of a three wheeled vehicle.*

System	Mass	CG (m, m, m)	Inertia (Kg- m^2) [I_{xx}, I_{xy}, I_{yz} I_{yx}, I_{yy}, I_{xz} I_{zy}, I_{zx}, I_{zz}]
Steering system (without front tire)	6.385 kg	−0.0033, 0.031,0.454 From center of the front wheel. Part coordinate system is rotated by 19 deg. about 'Y' axis.	[1.85, 0.0223, 0.0312 0.0223, 1.83, −0.0110 0.0312, −0.0110, 0.1060]
Front wheel	10.3 kg	0, −0.0073, 0 From center of the front wheel.	[0.1140, 0.0, 0.0 0.0 , 0.1840, 0.0

			0.0, 0.0, 0.1140]
Frame assembly	488.27	1.376, 0, 0.629 From center of the front wheel.	[3460.0, 0.0382, 0.0268 0.0382, 3670.0, 385.0 0.0268, 385.0, 2430.0]
Front suspension	3.76	−0.006, 0.041, 0.0824 From center of the front wheel.	[0.0557, 0.0003, 0.00155 0.0003, 0.0335, 0.0062 0.00155, 0.0062, 0.0439]
Rear suspension	15.70	−0.0968, 0.575, 0.060 From the rear wheel center.	[9.150, −0.1030, −0.0543 −0.1030, 9.360, 0.0874 −0.0543, 0.0874, 1.020]
Power train	58	0.127, 0.575, 0 From the left side rear wheel center.	[19.50, 0.0, 0.0 0.0, 20.60, −4.123 0.0, −4.123, 11.5]
Rear wheel	10.3	Center of the wheel	[0.1140, 0.0, 0.0 0.0 , 0.1840, 0.0 0.0, 0.0, 0.1140]

Table 3: *Mass and inertia properties of a three wheeled vehicle. The moment of inertia matrices are defined using part coordinate systems (see also [5]). The part coordinate systems themselves are defined relative to the global XYZ axes. Where no rotation is specified, the part coordinate system in the vehicle reference configuration (or initial assembly) is taken to be perfectly aligned with the global XYZ axes.*