Towards Long-Time Simulation of Soft Tissue Simulant Penetration

Ramsharan Rangarajan, Raymond Ryckman and Adrian Lew
Department of Mechanical Engineering, Stanford University, USA.
Email: {rram,rryckman,lewa}@stanford.edu

Abstract

We present a conceptual discussion of a numerical method to simulate the penetration of a ballistic gel by a projectile. The method draws on the computational advantages presented by asynchronous variational integrators (AVIs) and an immersed boundary method to result in an easily parallelizable, efficient and adaptive algorithm. The paper focuses on the algorithmic ideas involved– at the expense of details and a simplified model.

Figure 1: Snapshot of a cavity left in a ballistic gel by a bullet. Source:http://www.naaminis.com/pix/25gel02.jpg

1 Introduction

In this paper, we discuss some ideas we have for simulating the mechanics of penetration in a soft-tissue simulant. Such simulations are of immediate interest in evaluating firearms and bullets (see fig. 1) and in modelling the complex wounding process. Parameters such as the depth of penetration of a projectile, its trajectory or the extent of the resulting wound in the ballistic gel(tissue simulant) can be estimated to aid in optimizing armor materials and designs.

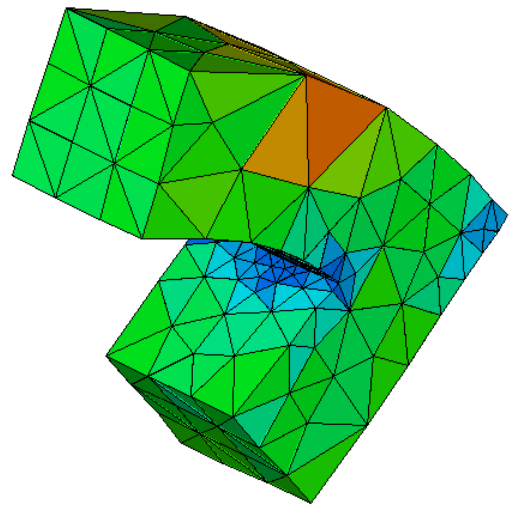
The numerical algorithm that we propose consists in adopting a finite element discretization of the gel, with an explicit asynchronous time stepping for the elements, a contact algorithm to handle the impact of the projectile and an adaptive remeshing technique. AVIs enable each element in the finite element mesh to be advanced independently (in time) with a time step befitting the local dynamics. As a result, small time steps can be adopted for elements close to the zone of impact or in regions of high stresses to resolve events occurring at small time scales.

The quality of the mesh used for the gel deteriorates as the projectile penetrates– a result of elements with large strains and poor aspect ratios. It is therefore necessary to adopt a fresh discretization for the gel. This is achieved with an adaptive remeshing algorithm based on an immersed boundary method– the boundary of the deformed gel is immersed in a simple discretization from which a new mesh for the gel, adapted to suit the local dynamics, is extracted. The displacement and velocity fields are used to dictate the element sizes in the new mesh. These fields are transferred to the new mesh and the simulation is set up to continue.

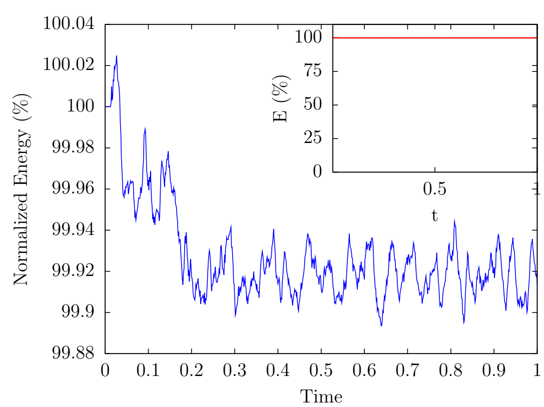
While discussing these new ideas and evaluating them using simple examples, we make a number of simplifying assumptions. While we will highlight them in the sections that follow, the most prominent among them is that we model the phenomenon as an impact of a *rigid* projectile on an *elastic* gel. Penetration is an extremely complex physical phenomenon heavily dependent on the materials involved in testing. Experiments, for instance firing high speed bullets at steel plates of different thicknesses (see [2]), reveal large deformations, high strain rates, plastic flow, fracture, temperature changes as well as micro-structural alterations such as formation of shear bands. It is not hard to imagine a transfer of mass (besides momentum) from the projectile to the target or even adhesion.

Utility of numerical simulations such as ours depends on how much of such physics we can model. But there are significant numerical challenges to be overcome. Identifying surfaces of contact, avoiding interpenetration of colliding bodies or handling geometries with sharp corners is still a cumbersome task in many state-of-the-art contact algorithms. There is still some way to go before numerics start yielding insight about the phenomenon or answering specific questions- such as the influence of surface roughness in such problems. In this respect, this article should be viewed as a progress report based on exciting ideas we have to address some of the computational issues arising in simulations of penetration.

In the following sections, we discuss the numerical algorithms we adopt to simulate a simplified penetration problem– an asynchronous variational time integrator, a con-

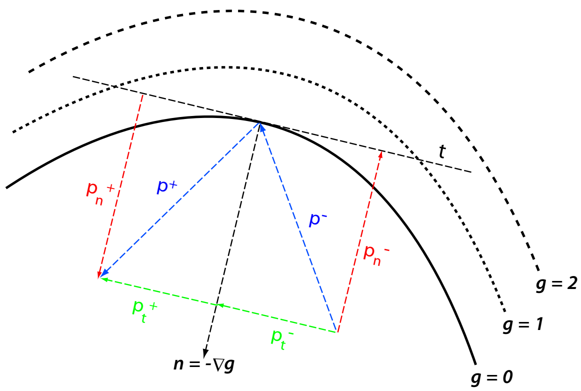


(a) Snapshot of a simulation of the impact of an ‘L’ shaped beam against a rigid wall (not shown) using AVI and the contact algorithm described next. Elements are colored according to the time step adopted, with blue representing an order of magnitude smaller than red. The computational efficiency of AVIs enables the resolution of high frequency modes seen in such problems

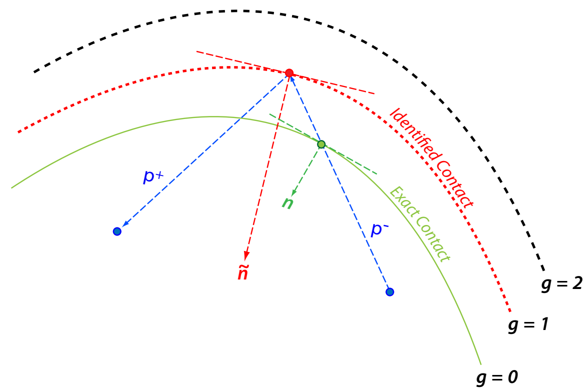


(b) Energy conservation characteristics of AVIs demonstrated in the impact problem. The energy is conserved nearly exactly, as the inset shows, with only a minor drop in value through impact, which can be regulated through the selection of the time step for the elements in contact.

Figure 2: AVI with contact for dynamics simulations.



(a) An idealization of the proposed contact algorithm. The contact constraint is given by the condition $g \leq 0$. When contact occurs ($g = 0$) with the rigid projectile, the normal component of the momentum of the gel is reversed while keeping the tangential component unaltered.



(b) Interpenetration is permitted in this algorithm. Even though contact may be detected after the gel has penetrated the projectile, this can be made negligible by adopting small time steps for elements close to the contact region.

Figure 3: Illustration of the contact algorithm.

tact algorithm and an adaptive remeshing strategy. We will focus on the main ideas rather delving into specifics, which can be found in the references provided.

2 Asynchronous Variational Time-Integrators

AVI possesses two properties most desirable in dynamic simulations,

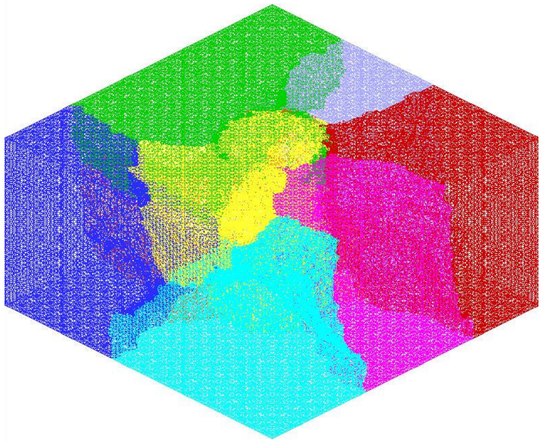
- being variational, it has outstanding momentum and energy conservation properties, and
- being asynchronous, it has the distinguishing feature of permitting an independent selection of time steps for each element in a mesh.

Fig.2 shows a snapshot of the simulation of the impact of an 'L' shaped beam against a rigid wall using AVI and the almost exact energy conservation of the system.

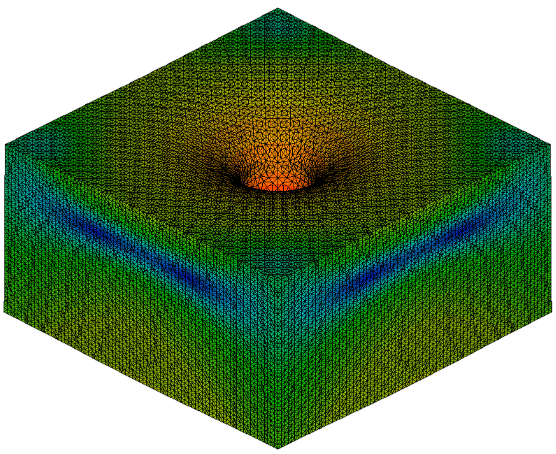
With traditional time integrators, the time step for the entire mesh is dictated by the minimum over the range of stable time steps for elements in the mesh. This can be particularly hindering in high velocity contact simulations. Local stiffening of the material and adaptively refined meshes can cripple long-time simulations. With AVI, each element's time step need only satisfy its own stability criterion, dependent on the local sound speed, material velocity and element size, rather than the most stringent one for the mesh. See [6] for a detailed derivation and analysis of the algorithm and [3] for implementation details.

3 Contact Algorithm

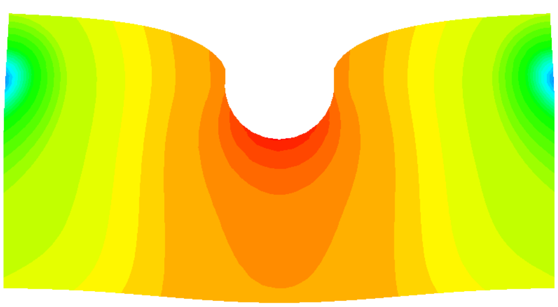
Next, we discuss a contact algorithm formulated to handle the impact of the projectile on the gel. The projectile is approximated as a rigid body and the gel as a soft hyperelastic material. The rigid body approximation is adopted mainly for the significant algorithmic simplifications it yields, though this can be loosely justified by the contrast in properties of the projectile and the gel.



(a) Partition of the domain for parallel processing.



(b) An elastic (Neohookean) block impacted by a rigid sphere.



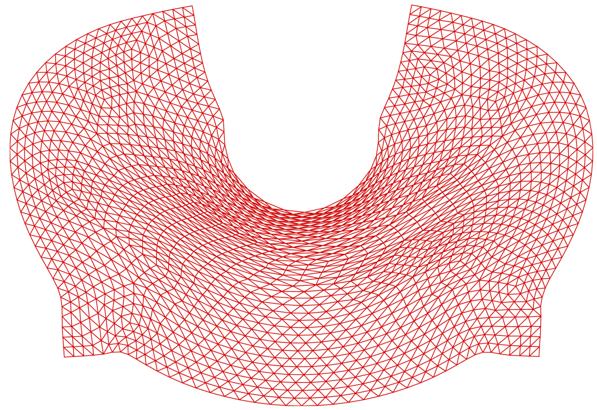
(c) A slice through the center of the block showing the large deformation. The color contours correspond to the magnitude of velocity, decreasing in magnitude from blue to red.

Figure 4: A snapshot of the impact of a rigid sphere on an elastic block modeled using the contact algorithm described.

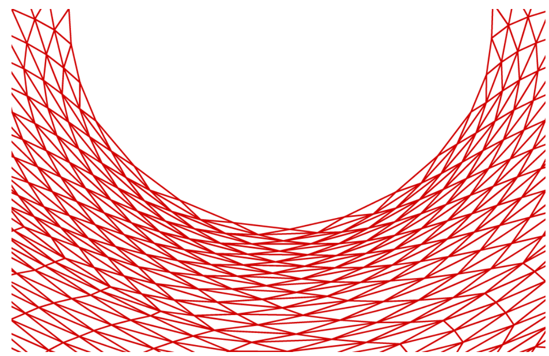
Traditionally, contact has been modeled as a *constrained optimization* problem in which an energy functional is rendered stationary subject to the constraint of no interpenetration. In the context of the current (discretized) problem, such a constraint can be expressed in the form $g(\mathbf{x}) \leq 0$, where $\mathbf{x}$ represents the set of nodal positions of the gel and g is a function that is strictly positive in the interior of the projectile, zero on its boundary and strictly negative in the exterior. Note that g is implicitly a function of time as well via the positions of the projectile and the gel.

With this description, collision occurs when $g(\mathbf{x}) = C \geq 0$. Upon detecting a collision (at the end of a time step of some element), the component of the momentum of the gel normal to the surface $g(\mathbf{x}) = C$ is reversed while leaving the tangential component unchanged, see fig. 3. Note that no projection onto the boundary $g(\mathbf{x}) = 0$ is performed to remove any penetration that has occurred, although this is one possibility. In this way, the gel effectively sees the contact "wall" wherever it falls at the end of a time step, instead of at a fixed location. An undesirable consequence of permitting such interpenetration is the dissipation of energy. However, the time steps for elements approaching contact with the projectile can be taken to be very small (leaving the time steps elsewhere unaltered) so that interpenetration and consequently the energy loss is small. The almost exact energy conservation in the example of the L-shaped beam shown above demonstrates precisely this. Another example of a rigid sphere impacting a block is shown in fig. 4.

In comparison with the above algorithm, an approach of computing the exact time of contact for each node close to the projectile would prove prohibitively expensive. A penalty approach is commonly used to minimize interpenetration, whereby the potential in the (interior of the) projectile is made large, approaching that of a rigid body, so that it is energetically unfavorable for the gel to cross the boundary of the projectile. While this has the advantage that an inequality constraint need not be explicitly handled, determining how stiff the penalty potential should be while maintaining accuracy and avoiding ill-conditioning issues is often non trivial. Likewise, Lagrange multipliers have also been used to impose the contact constraint at the cost of adding to the number of unknowns in the system, and making it implicit. A comprehensive review of contact algorithms can be found in [9, 10].



(a) Mesh undergoing large deformation.



(b) Close-up view of the elements in the region of contact.

Figure 5: A $2D$ example of a rigid ball impacting a square in which poorly shaped elements result from large deformations of the gel impacted by a rigid ball. This renders explicit time integration algorithms unstable unless extremely small time steps are used.

4 A Preliminary Penetration Algorithm With Adaptive Remeshing

In principle, the explicit time integration and contact algorithms are sufficient to simulate the long-term behavior of the dynamic system under consideration. However, as the simulation progresses, elements in the gel get significantly deformed resulting in poor aspect ratios, as shown in fig. 5. This necessitates a smaller time step to ensure the stability of the integrator and finally results in an unfeasible time step. The remedy lies in adopting a better discretization for the current configuration. This is precisely what we discuss next with the aid of a $2D$ example. Most of the ideas can be extended to the $3D$ case in a straightforward manner.

The adaptive remeshing algorithm broadly consists of the following steps:

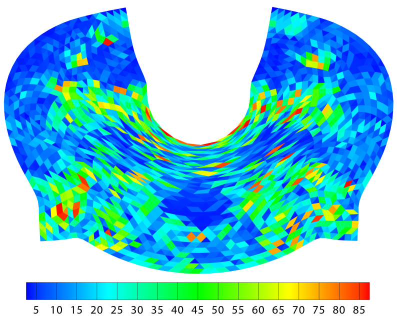
(a) determining when the quality of the mesh of the gel has deteriorated enough to warrant a new discretization,

(b) computing an indicator of how fine the mesh need locally be to represent all current velocity, strain stress and displacement fields,

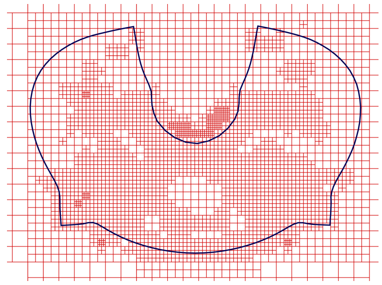
(c) creating a new mesh for the current configuration of the mesh with element sizes roughly as specified by the indicator and

(d) transferring fields (such as displacements and velocities) via interpolation to ones on the new mesh.

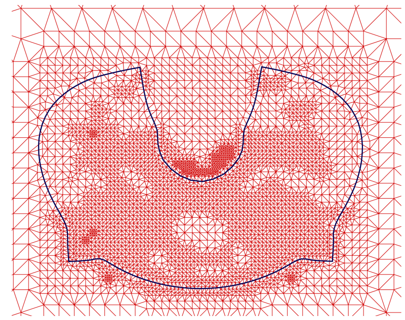
No doubt, each of the above steps is a difficult question and there have been numerous solutions proposed, each tailored to suit



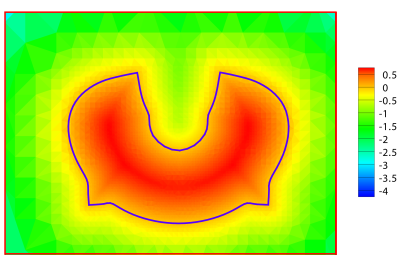
(a) We adopt an ad-hoc local refinement indicator based on identifying regions of high curvature in the displacement or velocity fields. Shown here is the curvature (in degrees) for the vertical component of the piecewise linear velocity field. Note that change in angles can be as large as 90°.



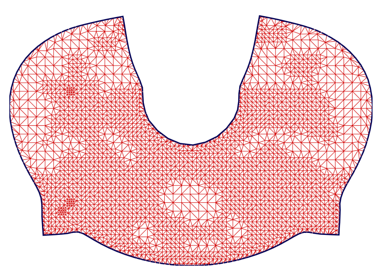
(b) The remeshing strategy involves the construction of a quadtree over a square domain encompassing the gel.



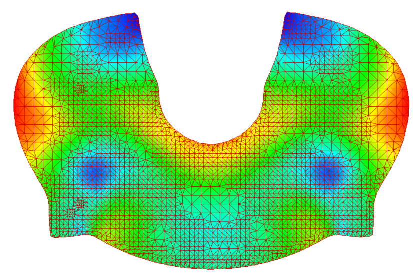
(c) A *Delaunay* triangulation is built over the convex hull of the centers of the quadtree nodes. Note that this triangulation is sufficiently refined in regions where the indicator is large in the domain of the gel.



(d) The gel is considered to be immersed in the triangulation of the square. To extract a mesh for the gel, the linear interpolant of the signed distance function to the boundary of the gel is computed over all nodes of the triangulation.



(e) A discretization for the gel is determined by trimming (and subdividing quadrilateral cuts) elements with the zero level-set of the linearized signed distance function.



(f) Displacement and velocity fields are transferred to the new mesh via interpolation. Shown here is the vertical component of the velocity field

Figure 6: Pictorial description of the adaptive remeshing algorithm. The distinguishing feature of this algorithm is its simplicity– it avoids many of the cumbersome operations (like edge flipping, mesh smoothing) commonly associated with meshing. It extends to $3D$ in a rather straightforward manner and can be automated quite easily.

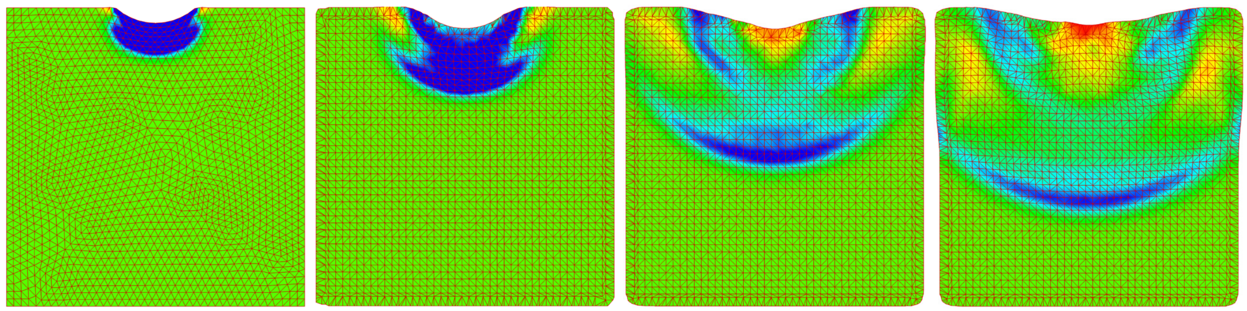


Figure 7: Deformation of a square block, impacted by a circular ball, computed using the remeshing strategy described. The block was remeshed four times (after fixed intervals) without considering adaptivity. Shown also are the contours of the vertical component of the velocity.

a particular application. Shown in fig. 6 is a pictorial description of the adaptive remeshing algorithm that we propose. The highlight is the *remeshing strategy* that draws from a discontinuous Galerkin based immersed boundary method [5, 8] we recently formulated for solid mechanics. The idea is to immerse the boundary of the domain to be meshed (the gel in this case) in a simple discretization and extract a mesh that approximately fits the boundary. In this way, especially in $3D$, the difficulties associated with automatic mesh generation are quite easily overcome. The rationale behind permitting an approximate mesh is that it would be futile to attempt a perfectly conforming discretization considering the numerical approximations involved and that the geometries are already only approximately represented. Moreover, the approximations in the domain are very small- decreasing quadratically with the new mesh size along the boundary of the gel. It is important to note that even though with this approach elements near the boundary are often smaller than those in the interior, thanks to AVI, they run with their own time step, without affecting the overall time step of the system.

The simulations shown above and in fig. 7 are only preliminary ideas. These have to still be refined and precisely analyzed to enhance the robustness of the procedure, the *Achilles heel* of every existing such approach.

5 Summary

Penetration problems are of great practical as well as academic interest, playing host to a whole range of physical phenomena. Numerical methods are a key tool to understanding the underlying physics. But there are significant challenges to be overcome to gain realistic insights (rather than being misled by algorithmic artifacts). The numerical method presented here is a step in this direction. Besides its computational efficiency and capability to simulate long-term dynamics, we also try to thoroughly understand the approximations involved. With the (afore mentioned) numerical tools, modern computational resources and more realistic models of the penetration phenomenon, simulations such as ours can be expected to reveal exciting details.

References

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